The Bank Liquidity Channel of Financial (In)stability*

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Abstract

We examine the system-wide effects of liquidity regulation on banks' balance sheets. In the general equilibrium model, banks have to hold liquid assets, and choose among illiquid assets varying in the extent to which they are difficult to value before maturity, e.g., structured securities. By improving the liquidity of interbank markets, liquidity requirements can induce banks to invest in such complex assets. We evaluate the welfare properties of combining liquidity regulation with other financial-stability policies, and show that it can complement ex-ante policies, such as asset-specific taxes, whereas it can undermine the benefits of ex-post interventions, such as quantitative easing.

Keywords: liquidity regulation, securitization, interbank markets, financial stability, quantitative easing

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1 Introduction

Banks' liquidity management takes the center stage in policy debates on financial stability. Their systemic importance as suppliers of liquidity to both the real and the remaining financial sector gives rise to the need for regulation with the goal of mitigating liquidity risk (e.g., Kashyap, Rajan and Stein, 2002; Gatev and Strahan, 2006; Acharya and Plantin, 2021). In addition, the liquidity composition of banks' balance sheets is a relevant determinant of monetary-policy transmission (see, among others Kashyap and Stein, 2000). The failure of some banks to preserve a level of liquidity that would allow them to shield their operations from disruptions due to bank funding shocks has prompted tighter liquidity regulation around the globe, in the form of the Liquidity Coverage Ratio (LCR).

A relevant consideration affecting the full impact of liquidity regulation is the way banks invest in assets that have limited eligibility for satisfying liquidity requirements. As securitization is an important channel through which banks seek to enhance their liquidity while accommodating risk taking in other asset classes, banks' investment in complex assets, such as structured securities, matters not only for their own solvency but also for other banks' ability to transfer credit risk. To shed light on the relationship between liquid and complex assets on the balance sheets in the banking system, this paper develops a general equilibrium model, and considers the effects of tighter liquidity regulation on banks' investment in complex assets, their provision of liquidity in the interbank market, and the implications for allocative efficiency arising from the interaction of liquidity regulation and other policies aimed at fostering financial stability.

In the model, banks maintain a required fraction of liquid assets, similar to the implementation of the U.S. Liquidity Coverage Ratio of 2013, which requires a subset of bank holding companies (BHCs) to hold an amount of high quality liquid assets (HQLA) that is sufficient to withstand their projected total net cash outflows over a 30-day period of significant stress.[1] They invest the remainder of their portfolios in long-term risky assets that differ only in terms of their complexity. Complex assets represent investments that are hard to value before maturity, such as structured financial products.[2] In contrast, simple assets are relatively easy to value and exhibit an earlier

[1]For example, the most liquid assets that can be used to satisfy the LCR without any discount include excess reserves, Treasury securities, government agency debt and MBS (not including government-sponsored agency debt and MBS), and sovereign debt with zero risk weights.

[2]Gorton and Metrick (2012) note that in the case of collateralized debt obligations, it is difficult to predict the payoff associated with each tranche. Additionally, Brunnermeier (2009) argues that the illiquidity of structured products during the crisis was associated with a loss of confidence in the ability to value these assets and in the reliability of ratings. For example, on August 9, 2007, BNP Paribas suspended valuations of three of its investment funds due to an inability to value assets that were exposed to the U.S. securitization market, eventually leading to a bank run on Northern Rock. Concerns about the quality

resolution of the uncertainty regarding their payoffs. Some fraction of depositors of each bank demand liquidity depending on their intrinsic needs as well as their confidence in their bank, which in turn can depend on the opacity of their bank's assets and the state of the economy. Banks with excess liquidity or shortfalls relative to this demand can then trade in an interbank market.

Motivated by the observed illiquidity of complex assets during the crisis (Gorton and Metrick, 2010), we show the existence of an equilibrium in which complexity has two important implications for bank performance and the pattern of interbank trading. First, it increases a bank's exposure to aggregate shocks, resulting in a procyclical quality of liquidity provision to depositors. In good times, which corresponds to states in which risky assets yield a high expected return, banks that invest in complex assets perform better on average because depositors, who cannot observe the quality of the complex assets for an individual bank but are confident in the expected return, maintain their investment until maturity. Banks that invest in simple assets perform worse on average because depositors run on the subset of banks whose assets are revealed to be of low quality.[3] In bad times, or crises, banks that invest in complex assets perform worse on average because uncertainty about the quality of their assets induces depositors to run. Banks that invest in simple assets perform better on average because depositors maintain their investments in the subset of banks whose assets are revealed to be of high quality.[4] Tables 1a and 1b show when banks invested in either type of asset experience a run.

Second, complex-asset holdings also increase a bank's capacity to respond to liquidity stress by selling its long-term assets on the interbank market. This is because the symmetric opacity associated with complex assets reduces asymmetric information and facilitates trade (Dang, Gorton and Holmström, 2010). However, if a bank invests in simple assets that turn out to be of low quality, then it cannot sell them to raise liquidity. In this manner, our model implicitly takes into account the possibility for banks to securitize their illiquid loans, thereby making them liquid (interbank loans), as their ability to do so is spurred by their investment in complex assets. Table 1c summarizes which type of asset performs better in different states.

We use the model to analyze how liquidity regulation affects the liquidity of interbank markets and banks' investment in complex assets. We illustrate channels by which

of securitized assets were ultimately associated with increasing costs of credit and declines in the real economy (Gertler and Gilchrist, 2018).

[3]This advantage of complexity in allowing banks to sell their assets at a deterministic rate is consistent with the advantages of a late resolution of uncertainty described in Hirshleifer (1971).

[4]Note that complex assets are not assumed to have procyclical inherent risk compared to simple assets. Their relatively procyclical character is solely due to how their opacity interacts with depositor sentiment in good versus bad times.

Table 1: This table indicates when a run occurs for a bank invested in complex assets (Table 1a) or simple assets (Table 1b). It also shows which assets perform better based on the individual return and aggregate state (Table 1c).

(a) Complex assets run states

		Individual return	
		High	Low
State	Good		
	Bad	X	X

(b) Simple assets run states

		Individual return	
		High	Low
State	Good		X
	Bad		X

(c) Advantageous asset by state

		Individual return	
		High	Low
State	Good	Neither	Complex
	Bad	Simple	Complex

tighter liquidity regulation can either substitute for or complement investment in complex assets. On the one hand, requiring banks to hold greater liquidity buffers reduces the liquidity advantage of complexity in good times. On the other hand, it also increases the supply of liquidity in bad times, which leads to an increase in asset prices. Higher anticipated asset prices partially insure banks against runs associated with complex assets, which encourages greater ex-ante investment in the latter.[5]

To the extent that the availability and use of securitization, fitting our description of complex assets, has enabled lending to subprime borrowers, which is seen as a key precursor to the financial crisis (Mian and Sufi, 2009), our main result points to a potentially destabilizing effect on the financial system as an unintended consequence of liquidity regulation. Through the lens of our model, we then explore how liquidity regulation can be combined with other policies to counteract this effect and foster financial stability. Liquidity regulation can be used to complement ex-ante financial-stability policies, such as asset-specific taxes. In particular, the equilibrium degree of investment in complex assets is generically inefficient because the interbank market provides incomplete insurance, resulting in a distortionary pecuniary externality. Liquidity requirements determine how the equilibrium investment in complex assets compares to the level chosen by a constrained planner.[6] The constrained-efficient investment in complex assets can be

[5]In a simulation of the model motivated by the Great Financial Crisis, tighter liquidity regulation has a net positive effect on banks' investment in complex assets, which in turn dampens the effect of liquidity regulation in supporting asset prices during crises.

[6]To be more precise, on the one hand, the planner may have a stronger incentive to invest in complex assets compared to the individual banks because it internalizes the full return of these assets in the bad

induced via asset-specific taxes, but whether simple or complex assets should be taxed depends on the tightness of liquidity requirements.

As liquidity regulation affects banks' liquid-asset portfolio and their willingness to provide funds in the interbank market, the liquidity of which determines the pass-through of monetary-policy rates to interbank rates (see, e.g., Bianchi and Bigio, 2022), our model also links to monetary-policy transmission. Given central-bank purchases of illiquid assets in the course of quantitative easing (QE), we zoom in on the interaction between liquidity regulation and QE, which are concurrently implemented policies not only in the U.S. but also in the euro area. We show that tighter liquidity regulation can undermine the benefits of ex-post policies such as QE, i.e., asset purchases by the government in bad times. QE leads to higher asset prices to support solvent but illiquid banks, but it also involves a cost since the bond purchases must be financed with taxes. If QE is unanticipated, then it always improves welfare. However, if QE is anticipated, then banks respond by shifting their portfolios towards complex assets ex ante, which has an offsetting negative effect on the complex-asset price. Because of this attenuation, the gains from QE may wind up too small relative to its financing costs.[7]

Relation to the literature. In this paper, we set out to analyze how liquidity regulation affects banks' balance sheets, in particular the composition of illiquid assets that are not eligible to satisfy liquidity requirements imposed by rules such as the Liquidity Coverage Ratio. We further consider how this channel influences the effect of liquidity regulation on interbank debt markets, welfare, and the effectiveness of financial-stability policies in a general equilibrium model.

Allen and Gale (2017) provide a survey of the literature on liquidity regulation. They remark that there is little consensus regarding the specific nature of the market failures that it is intended to target. For example, liquidity regulations have been motivated on the basis of correcting for fire-sale externalities in short-term funding markets (Perotti and Suarez, 2011) as well as incomplete information of depositors about a bank's vulnerability to a run (Diamond and Kashyap, 2016). Dewatripont and Tirole (2018) analyze inconsistent shocks and interactions between liquidity regulation and solvency

state, whereas the individual banks that invest in complex assets receive only a fraction of this return based on the interbank market price. On the other hand, the planner may have a stronger incentive to invest in simple assets because it internalizes that this would effectively distribute more liquidity to the liquidity-shocked depositors of the distressed banks. If liquidity requirements are sufficiently tight, each safe bank has a large amount of excess liquidity that can be used to buy assets from the distressed banks, and the latter effect dominates.

[7] This result is consistent with the idea that bailout expectations allow banks to finance risky projects with effectively subsidized debt (see Atkeson et al., 2019; Duffie, 2019, for evidence on this distortion), and they, thus, do not fully internalize the social cost of bank failure.

concerns.[8]

Our paper also contributes to a strand of the literature on policy interventions that are meant to support banks during crises, in particular quantitative easing. A natural connection to our model arises from the fact that, as pointed out by Chakraborty, Goldstein and MacKinlay (2020), quantitative easing interacts directly with banks' complex-asset holdings, e.g., structured securities, as the latter were targeted during two rounds of asset purchases in the U.S. Holmström and Tirole (1998) argue that government interventions to actively manage liquidity supply can be welfare improving when liquidity shocks are correlated. However, Farhi and Tirole (2012) show that the anticipation of bailouts can induce banks to take excessive correlated risks, although Philippon and Wang (2022) argue that restricting bailouts to better-performing banks can ameliorate moral hazard incentives. Acharya and Rajan (2022) show that on- and off-balance sheet adjustments can hinder the effectiveness of liquidity injections by the central bank. Besides considering how liquidity regulation interacts with QE, we also consider ex-ante financial-stability policies such as asset-specific taxes. We show that they can be used to implement a constrained-efficient level of investment in complex assets. As such, our paper relates to DiTella (2019) and the characterization of optimal financial-regulation policy therein, showing that the socially optimal allocation can be implemented with a tax on asset holdings internalizing hidden-trade externalities.

Our paper also relates to the literature on bank opacity, particularly that driven by securitization, which focuses on different elements: endogenous information production in financial markets (Glode, Opp and Zhang, 2018; Azarmsa and Cong, 2020), amplification channels and systemic risks (Ibragimov, Jaffee and Walden, 2011), the disclosure of bank-specific information in good and bad times by the regulator (Bouvard, Chaigneau and de Motta, 2015; Goldstein and Leitner, 2018), the signalling of asset quality by banks (Chemla and Hennessy, 2014), a trade-off between transparency and risk diversification (DeMarzo, 2005; Duffie, 2008), or banks' choice of opacity and financial

[8] In terms of the empirical documentation of the effects of the LCR or very similar policies on banks' asset portfolio and interbank markets, Banerjee and Mio (2018) show that liquidity regulation in the UK led to higher investment in liquid assets and reduced reliance on short-term intra-financial loans and wholesale funding. Bonner and Eijffinger (2016) document that liquidity regulation in the Netherlands led to increased demand for long-term interbank loans. In the U.S., the LCR has been associated with reduced liquidity creation and fire-sale risk (Roberts, Sarkar and Shachar, 2018), as well as a greater degree of risk taking for banks with a higher share of stable liabilities (Bosshardt, Kakhbod and Saidi, 2022). Afonso et al. (2020) argue that liquidity regulations may have increased banks' desired level of reserves, potentially contributing to the high volatility in the U.S. repo market in September 2019. BIS (2017) argues that the LCR may lead to segmentation in repo markets by increasing the demand for trades that allow banks to maintain their regulatory ratios. In contrast to these existing findings, we present a model that rationalizes the possibility that the LCR has increased banks' ability to invest in complex, hard-to-value assets.

crises (Babus and Farboodi, 2020), among others. In contrast to this important line of work, we study under what conditions tighter liquidity requirements substitute for or complement banks' investment in complex assets, such as structured securities. Specifically, the degree of "complexity" in bank portfolios in our setting is somewhat related to the seminal models in Dang, Gorton and Holmström (2010) and Gorton and Ordoñez (2014). Focusing on optimal security design, Dang, Gorton and Holmström (2010) show that debt is welfare maximizing and information insensitive, and can give rise to crises. This is because when a bad systemic shock occurs, information-insensitive securities become more sensitive to information acquisition. Gorton and Ordoñez (2014) find that debt contracts that are insensitive to information result in a surge of credit, thereby promoting an upswing in financial vulnerability over a period of time as businesses having low-quality collateral become eligible to borrow. However, when unfavorable news is disclosed about the collateral value, lenders are induced to obtain information, which eventually leads to an abrupt reduction in output. In contrast, in our model there is no information acquisition. More than that, simple and complex securities are identical ex ante, and neither banks nor depositors obtain any information about complex assets prior to their maturity. Most importantly, rather than on security design, we focus on how liquidity regulation interacts with banks' complex-asset holdings that are associated with greater informational uncertainty, with crucial repercussions for interbank markets, welfare, and monetary-policy transmission (quantitative easing in particular).

Finally, our model's implications regarding fire sales in the interbank market link to other theories of asset sell-offs during financial crises (e.g., Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997; Shleifer and Vishny, 1997). Fire sales can be exacerbated by predatory trading (Brunnermeier and Pedersen, 2005). In addition, a run-up in either the repo or asset-backed commercial paper market can occur due to an increase in "money demand" (Gorton and Metrick, 2012) or global imbalances (Caballero and Krishnamurthy, 2009). Adverse selection can also lead to fire sales in the interbank market. For example, under adverse selection in secondary debt markets (Gorton and Pennacchi, 1990), costly information acquisition (Ahnert and Kakhbod, 2018) and information production may be destabilizing (Dang, Gorton and Holmström, 2010; Gorton and Ordoñez, 2014). In contrast to these models, our fire-sale mechanism hinges on interactions between liquidity requirements and banks' choice to invest in complex or simple assets.[9]

[9]More generally, our model is related to papers on financial crises, which the literature has argued to result from either weak fundamentals or panics (Goldstein, 2012). A self-fulfilling crisis can be caused by a panic among bank depositors, as in Diamond and Dybvig (1983), or among currency speculators, as in Obstfeld (1996). By contrast, fundamentals-based crises are analyzed by Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998), and Baron, Verner and Xiong (2021) for

2 Model

This section introduces a model in which liquidity risk and liquidity regulation affect a bank's incentive to invest the portion of its portfolio that is ineligible for satisfying liquidity requirements in either complex or simple assets. In the following sections, we then characterize the equilibrium, and illustrate channels by which tighter liquidity requirements affect asset prices and investment in complex assets. We also show that the equilibrium investment in complex assets can be either excessive or insufficient depending on the tightness of liquidity requirements. Finally, we examine how liquidity requirements interact with other financial-stability policies.

Overview. There are three periods, $t \in \{0,1,2\}$, and a mass one of limited-liability banks indexed by $i \in [0,1]$. At $t = 0$, each bank acquires funding from a mass one of depositors that each deposit one unit of capital. Liquidity regulations require banks to hold a fraction of their assets in liquid investments. Banks can invest their remaining assets in long-term, risky investments of varying complexity. At $t = 1$, the publicly observed economic state ω is realized as either good, $\omega = g$, or bad, $\omega = b$. It is commonly known that the good state is realized with probability η. Subsequently, some depositors may withdraw early. Each bank may then have an excess or a shortfall of liquidity that it can trade against in an interbank market. At $t = 2$, asset returns are realized, interbank trades are completed, and banks distribute any profits back to their depositors.

Depositors. There are two types of depositors. A *normal (late)* depositor has a constant marginal utility of 1 for all payoffs, regardless of when they are received. A *liquidity-shocked (early)* depositor experiences a liquidity shock at period 1, which is represented by having a marginal utility that is equal to $\alpha > 1$ for the first $\kappa > 1$ units of capital received in period 1, and that is equal to 1 for capital in excess of κ in period 1 or any payoffs received in period 2. Similarly to Dang, Gorton and Holmström (2010), a depositor's utility $U(x)$ in period 1 from consuming x can be summarized as follows:

$$U(x) = x\mathbf{1}_{\text{shocked}}\left(\alpha\mathbf{1}_{\{x\leq\kappa\}} + \mathbf{1}_{\{x>\kappa\}}\right) + x\mathbf{1}_{\text{normal}}.$$

Each depositor's type is private information, but the fraction of liquidity-shocked depositors $\phi \in (0,1)$ is publicly known.

banks, and by Krugman (1979) for currency crises. Both of these views have also been considered in global coordination games by Morris and Shin (1998) and Corsetti et al. (2004) for currency attacks, and by Morris and Shin (2004) and Corsetti, Guimarães and Roubini (2006) for debt crises.

Liquidity regulation. In period 0, each bank must invest a fraction L of its assets in liquid investments that can be used to satisfy a regulatory liquidity requirement. In period 1, a bank can use its liquid assets to pay depositors who withdraw early.[10] We assume $L > \kappa\phi$ to ensure that a bank has sufficient liquidity to meet the liquidity needs of the liquidity-shocked depositors. Liquid assets that are held until period 2 yield a return that is normalized to 1.

A bank's portfolio choice. The remaining fraction $1 - L$ can be invested in long-term, risky investments that mature in period 2. There are two types of investments that we denote by θ. Complex assets, denoted by $\theta = C$, represent investments the quality of which is relatively difficult to evaluate before maturity, such as securitized assets and structured financial products. Simple assets, denoted by $\theta = S$, represent investments the quality of which can be evaluated relatively easily before maturity. Specifically, the returns for simple assets become public knowledge in period 1, whereas the returns for complex assets are not known until they mature in period 2. The two types of investments have identical return distributions that depend on the realization of the economic state ω. Specifically, both yield a return of $R > 0$ with probability μ_ω (depending on the economic state) and 0 otherwise, where $\mu_g > \mu_b$.

Endogenous choice of transparency. Any bank can choose at date $t = 0$, by deciding on its investment in either complex or simple assets, whether the return on its long-term, risky assets will become public knowledge in period 1 or not. Banks also have the option to invest their entire portfolio in liquid assets. A bank determines its portfolio so as to maximize the expected utility of its depositors.[11] A bank's portfolio choice is publicly observed. Henceforth, we refer to banks invested in simple, complex, or liquid investments as simple, complex, or liquid banks, respectively.

For simplicity, we rule out mixed portfolios. This assumption, however, does not affect our main conclusions, and the comparative statics of our model are qualitatively robust to relaxing it (see Online Appendix A). This is in part because if the returns to complex and simple assets are independent, complex banks are indifferent between them conditional on their run conditions,[12] which we introduce below.

[10] This is consistent with the guidance for the implementation of the Liquidity Coverage Ratio articulated in Basel Committee on Banking Supervision (2013), which states that firms may temporarily break the requirement during periods of financial stress.

[11] Each bank can be understood as being mutually owned by its depositors, as in Diamond and Dybvig (1983).

[12] Therefore, complex banks have no incentive to deviate. The incentive of simple banks to deviate depends on the level of complexity that is sufficient to trigger a run in bad times. They may have no

Debt contract. In period 0, each bank promises to pay $R_{d,i}$ to depositors that withdraw early in period 1, assuming it can meet the demand for liquidity. In period 2, the bank pays the remaining value of its assets to depositors that withdraw late. If the bank cannot meet the demand for liquidity in period 1, then it is said to experience a run. Specifically, the bank is liquidated in period 1, and each depositor receives a return in proportion to the bank's total value after liquidation.[13] Any remaining long-term assets that are not sold in the interbank market are liquidated and yield a return of zero. A bank chooses its early repayment to maximize the expected utility of its depositors.

Interbank market. In period 1, an interbank market allows banks with insufficient liquidity relative to the demand from early depositors to sell their long-term assets to banks with excess liquidity.

For convenience of notation, define a *normalized unit* of complex assets as the amount that yields an expected payoff of 1. In particular, a normalized unit of complex assets is equal to $\frac{1}{\mu_\omega R}$ units of complex assets. Denote the state-dependent price for a normalized unit of complex assets by $P_C(\omega)$. Similarly, a normalized unit of simple assets with a high return is equal to $\frac{1}{R}$ units of simple assets. Denote the price for a normalized unit of simple assets with a high return by $P_S(\omega)$. Note that simple assets with a low return cannot be sold since they are publicly observed to be worthless. Normalized units will be implicitly assumed for the rest of the paper.

The pattern of trade is as follows. If the mass of withdrawals in period 1 for bank i is equal to $\alpha_i(\omega)$, then the bank's net liquidity position in period 1 is given by $y_i(\omega) = L - \alpha_i(\omega)R_{d,i}$. If a bank has a liquidity shortfall, i.e., $y_i(\omega) < 0$, then it would like to sell $\frac{-y_i(\omega)}{P_\theta(\omega)}$ of its assets to generate enough liquidity to avoid a run. However, a bank can only sell up to $1 - L$ units of long-term assets, which corresponds to $\mu_\omega R(1 - L)$ normalized units of complex assets or $R(1 - L)$ normalized units of simple assets. A bank's supply of assets on the interbank market can be summarized by

$$S_{B,i}(P_C(\omega)) = \left[\frac{-y_i(\omega)}{P_C(\omega)} \wedge \left(\mathbf{1}_{\{\theta_i = C\}}\mu_\omega R + \mathbf{1}_{\{\theta_i = S \text{ \& } R_i = R\}}R\right)(1 - L)\right]^+,$$

where $A \wedge B$ denotes $\min\{A, B\}$ and $[A]^+$ denotes $\max\{0, A\}$.

If a bank has excess liquidity, i.e., $y_i(\omega) > 0$, then its demand for long-term assets

incentive to invest in complex assets, especially if there are any fixed costs associated with investing in another asset class. Even if they do invest in complex assets to a modest degree that does not trigger a run in the bad state, we have verified numerically that the comparative statics are similar.

[13] It is mathematically equivalent to alternatively suppose that each depositor receives R_d with a uniform probability that depends on the bank's value after liquidation.

depends on how the return compares to the return of 1 on its liquid assets. Specifically, in the market for long-term assets of type θ, the bank fully invests in $\frac{y_i(\omega)}{P_\theta(\omega)}$ normalized units if $P_\theta(\omega) < 1$, it is indifferent if $P_\theta(\omega) = 1$, and it will hold on to its liquid assets if $P_\theta(\omega) > 1$. A bank's demand can thus be summarized by

$$D_{\theta,i}(P_\theta(\omega)) = \mathbf{1}_{\{P_\theta(\omega)<1\}} \frac{y_i(\omega)}{P_\theta(\omega)} + \mathbf{1}_{\{P_\theta(\omega)=1\}} [0,\, y_i(\omega)],$$

where $[0,\, y_i(\omega)]$ indicates the respective range as the bank is indifferent between investing any amount up to $y_i(\omega)$ if $P_\theta(\omega) = 1$.

The price is determined by the market-clearing condition:

$$\int D_{\theta,i}(P_\theta(\omega))di = \int S_{\theta,i}(P_\theta(\omega))di.$$

Note that the interbank market can also be interpreted as a repo market with a haircut of $h_\theta(\omega) = 1 - P_\theta(\omega)$, where in the repo-market interpretation $P_\theta(\omega)$ represents the price of a bond backed by assets of type θ. See Online Appendix B for details.

3 Simplified Model: Exogenous Portfolio Choice

To facilitate the exposition, we first consider a simplified version of the model in which banks hold no excess liquidity and the fraction or "volume" of complex banks, which we denote as V, is exogenous. This simplified version illustrates the relationship between the debt contract, bank runs, and the price for complex assets in the interbank market, which is an important building block for solving the full model with endogenous V in Section 4. It can also be interpreted as a "short-run" version of the model capturing the direct effect of liquidity requirements on interbank markets before banks have time to adjust their portfolios. All proofs are relegated to the Appendix.

Proposition 1 (Simplified equilibrium)**.** *There exists an equilibrium in which the following hold:*

1. *Banks pay depositors that withdraw early a return of $R_d = \kappa$.*
2. *Liquidity-shocked depositors always withdraw early, and normal depositors withdraw early if and only if*
   - *the bank is complex and the economic state is bad.*
   - *the bank is simple and its individual return is low.*

3. *The price for simple assets is $P_S^*(\omega) = 1$, the complex-asset price in good times is $P_C^*(g) = 1$, and, for V sufficiently large, the complex-asset price in bad times related to the volume of complex banks by the corresponding interbank-market clearing condition:*

$$\underbrace{VR(1-L)\mu_b}_{\text{complex-asset supply}} = \underbrace{\Big(1-V\Big)\,\mu_b\,\frac{L-\kappa\phi}{P_C^*(b)}}_{\text{complex-asset demand}} \tag{1}$$

4. *For suitable levels of V, the complex-asset price in bad times satisfies $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$.*

Banks' debt contract choice. Banks optimally pay a return of $R_d = \kappa$ to depositors that withdraw early because the elevated marginal utility $\alpha > 1$ of liquidity-shocked depositors creates an incentive to provide their full liquidity need κ.[14]

Depositors' withdrawal timing choice. There is an equilibrium in which liquidity-shocked depositors always withdraw early as long as the urgency for liquidity α is large enough relative to the maximal return on long-term assets R. Additionally, the normal depositors withdraw under the described conditions as long as the expected return for complex banks, $L + \mu_\omega R(1-L)$, exhibits a large enough disparity between the bad state and the good state. The proof of Lemma 1 in the Appendix provides further details on the joint determination of the banks' debt contract choice and the depositors' withdrawal timing choices.

Interbank market equilibrium. The supply of simple assets is always equal to zero. This is because only simple banks with a low return experience a run, but they cannot sell their observably worthless assets. Therefore, the price is at the maximum possible level of 1, at which the return is the same as that of liquid assets.

For complex assets, the price depends on the economic state. In good times, the supply is equal to zero since complex banks do not experience a run. Therefore, the price is at the maximum possible level, $P_C^*(g) = 1$. In bad times, complex banks experience a run and need to raise funds by selling their assets. At the same time, simple banks with a positive individual return have excess liquidity. The complex-asset price in bad times and the volume of complex banks are then inversely related based on the market-clearing condition shown in (1).[15] In particular, the equilibrium price $P_C^*(b)$ can be less than 1 if the volume of complex banks V is sufficiently large.

[14] See the proof of Lemma 1 in the Appendix for details.
[15] See proof of Lemma 2 for details.

Takeaways. The simplified version of our model has two important takeaways. First, it shows that simple banks default when their individual return is low, whereas complex banks default when the aggregate state is bad. Second, it shows that an active interbank market only exists for complex assets in the bad state, and characterizes the complex-asset price in equation (1). The default conditions and accessibility of liquidity via the interbank market determine the relative costs and benefits of investing in complex assets, which are used in the following section to endogenously determine the volume of complex banks.

4 Full Model: Endogenous Portfolio Choice

In order to analyze the effects of liquidity regulation and other financial-stability policies, it is important to take into account how banks respond by adjusting their asset portfolios. Building on the results from Section 3 regarding the performance of complex and simple banks, this section describes the full equilibrium featuring endogenous determination of the volume of complex banks, which we denote as V^*.

4.1 Equilibrium Description

Proposition 2 (Equilibrium)**.** *There exists an equilibrium in which:*

1. *Properties 1-3 of the equilibrium from Proposition 1 are satisfied.*
2. *All banks invest in long-term assets without any excess liquidity.*
3. *At the equilibrium volume of complex banks V^*, the complex-asset price in bad times satisfies $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$.*

Banks' portfolio choice. Banks optimally invest in one of the two types of long-term assets as long as the return for long-term assets R is sufficiently high.[16] Since banks are ex-ante identical and both types of risky assets are held in equilibrium (cf. the proof of Lemma 5 in the Appendix), the degree of investment in complex assets is determined by the condition that banks are indifferent between investing in complex and simple assets. Note that we can equivalently think of this condition as determining the equilibrium price for complex assets in the bad state, which is monotonically related to the volume

[16]Note that the proof shows not only that holding only liquid assets is a dominated portfolio, but also that banks have no incentive to hold excess liquidity conditional on investing in either complex or simple assets and maintaining a debt contract of $R_d = \kappa$.

of complex banks via the interbank-market clearing condition in (1). Given the debt contract, $R_d = \kappa$, and bank-run conditions as described in Proposition 1, the expected utility from investing in complex assets as a function of $P_C(b)$ can be written as

$$\begin{aligned}\mathbb{E}[U_C|P_C(b)] = \eta &\Bigg(\underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + \mu_g R(1-L)}_{\text{return to normal dep.}} \Bigg) \\ &+ (1-\eta) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \Bigg(\underbrace{L + P_C(b)\ \mu_b\ R(1-L)}_{\text{liquidation value}} \Bigg), \end{aligned} \tag{2}$$

and the expected utility from investing in simple assets can be written as

$$\begin{aligned}\mathbb{E}[U_S|P_C(b)] = \eta &\Bigg(\mu_g \Big(\underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + R(1-L)}_{\text{return to normal dep.}} \Big) \\ &+ (1-\mu_g) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \underbrace{L}_{\text{liquidation value}} \Bigg) \\ &+ (1-\eta) \Bigg(\mu_b \Big(\underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{\frac{\overbrace{L-\kappa\phi}^{\text{buy complex assets}}}{P_C(b)} + R(1-L)}_{\text{return to normal dep.}} \Big) \\ &+ (1-\mu_b) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \underbrace{L}_{\text{liquidation value}} \Bigg), \end{aligned} \tag{3}$$

where the blue terms correspond to cases where there is no bank run and the red terms correspond to cases where there is a bank run.

The relative benefit of investing in complex assets is then given by subtracting (3) from (2):

$$\begin{aligned}\Delta(P_C(b)) &\equiv \mathbb{E}[U_C|P_C(b)] - \mathbb{E}[U_S|P_C(b)] \\ &= \eta\mu_g * 0 \\ &\quad + \eta(1-\mu_g)\phi(\kappa - L)(\alpha - 1) \\ &\quad + (1-\eta)\mu_b \left[(\alpha\phi + 1 - \phi)(L + P_C(b)\mu_b R(1-L)) - \left(\alpha\kappa\phi + \frac{L-\kappa\phi}{P_C(b)} + R(1-L) \right) \right] \\ &\quad + (1-\eta)(1-\mu_b)(\alpha\phi + 1 - \phi)P_C(b)\mu_b R(1-L). \end{aligned} \tag{4}$$

The intuition is as follows. The first line of (4) reflects the fact that conditional on drawing a high return in the good state, complex and simple banks both achieve the same utility.

The second line reflects the fact that conditional on drawing a low return in the good state, complex banks achieve a higher utility because they can still service the full liquidity need of the liquidity-shocked depositors in period 1, whereas simple banks experience a run.

The third line reflects the fact that conditional on drawing a high return in the bad state, simple banks achieve a higher utility because they can service the full liquidity need of the liquidity-shocked depositors, earn a return on asset purchases from the interbank market, and accrue the return on its long-term assets, whereas complex banks experience a run.

The fourth line reflects the fact that conditional on drawing a low return in the bad state, complex banks achieve a higher utility because they can sell assets to reduce the liquidity shortfall in a run.

Table 1c summarizes which asset has an advantage depending on the individual return and aggregate state.

The equilibrium complex-asset price is determined by equating the net advantage of simple banks in the bad state to the net advantage of complex banks in the good state. Then, given the equilibrium complex-asset price, the equilibrium volume of complex banks V^* can be obtained via the interbank-market clearing condition in (1).

Interbank market. The feature that the asset price is lower in bad times compared to good times, or $P_C^*(b) < 1 = P_C^*(g)$, is consistent with the drop in asset prices that was observed during the Great Financial Crisis (Gorton and Metrick, 2012), but also with the idea that complex banks made use of securitization to increase their liquidity during the run-up to the crisis.

4.2 The Effect of Tightening the Liquidity Requirements

This subsection illustrates channels by which tighter liquidity requirements affect the equilibrium complex-asset price and the degree of investment in complex assets.

First, note that the effect of tightening liquidity requirements on the equilibrium complex-asset price in bad times is inversely related to its effect on the incentive to invest in complex assets. In particular, if tightening liquidity requirements decreases the incentive to invest in complex assets, then the price must increase to restore the indifference between investing in complex and simple assets in equilibrium.

To elaborate, recall the relative advantage of complex assets Δ as summarized by equation (4). Differentiating with respect to the liquidity requirement L at the equilibrium price $P_C^*(b)$ obtains:

$$\frac{\partial \Delta}{\partial L} = -\eta(1-\mu_g)\phi(\alpha-1) + (1-\eta)\mu_b(P_C^*(b)R-1)\left[-(\alpha\phi+1-\phi)+\frac{1}{P_C^*(b)}\right]. \tag{5}$$

The first term $-\eta(1-\mu_g)\phi(\alpha-1) < 0$ reflects the fact that tightening liquidity requirements reduces complex banks' superior ability to provide liquidity to early depositors, by increasing the liquidity that simple banks with a low return can distribute back to investors when they experience a run.

The second term corresponding to the bad state has two subterms with opposite signs. Recall that $P_C^*(b)R > 1$ (see Proposition 2). The first subterm, $-(\alpha\phi+1-\phi)$, is negative and reflects the fact that tightening liquidity requirements mitigates the advantage of complex banks relative to simple banks that draw a low return, which is their ability to mitigate runs by selling their long-term assets. Like in good times, tighter liquidity requirements lead to an increase in the liquidity that simple banks with a low return can distribute back to investors when they experience a run.

The second subterm, $\frac{1}{P_C^*(b)}$, is positive and reflects the fact that higher liquidity mitigates the disadvantage of complex banks relative to simple banks that draw a high return, which is their proneness to runs and subsequent inability to survive until period 2 to accrue the yield on their long-term assets. This is because tighter liquidity requirements lead to a reduction in the fraction of long-term assets that simple banks can invest in.

For η sufficiently large, the first term of $\frac{\partial \Delta}{\partial L}$ corresponding to the good state dominates, in which case increasing liquidity requirements must result in a reduction in the incentive to invest in complex assets regardless of which subterm dominates in the bad state. This determines a sufficient condition under which the net effect of tightening liquidity requirements on the complex-asset price is positive. In Online Appendix G.1, we present supporting evidence that tighter liquidity requirements, in the form of the Liquidity Coverage Ratio, are associated with higher complex-asset prices in bad times.

Proposition 3. *If η is sufficiently large, the equilibrium complex-asset price in bad times $P_C^*(b)$ is increasing in the liquidity level L.*

The change in the complex-asset price is mediated by two mechanisms. First, liquidity requirements reduce the complex-asset supply for each individual complex bank while increasing the aggregate supply of liquidity, which directly increases the complex-asset price, which follows from (1) while holding V constant. Second, depending on how this direct effect compares to the change in the price that is required to maintain indifference between investing in the two types of long-term assets, banks shift either towards or away from complex assets ex ante, which in general can lead to either a dampening or amplification of the price response.

In simulations, we find that banks shift towards complex assets as liquidity requirements tighten (for example, see the first column of Figure E.1 in Online Appendix E). In Online Appendix G.2, we present supporting evidence that tighter liquidity requirements following the LCR are associated with a greater degree of investment by U.S. banks in complex assets, specifically structured financial products, asset-backed securities, and mortgage-backed securities other than those that are counted as level 1 liquid assets satisfying the LCR without any discount.

4.3 Planner Solution

We next show that the equilibrium is generically inefficient, and that the pattern of inefficiency is monotonically related to the tightness of liquidity requirements.

Consider a regulator whose objective is to choose the volume of complex banks, denoted by V^W, to maximize the welfare in the economy, which is defined as the expected utility of depositors. The regulator is constrained to choices for which there is an equilibrium in which the privately optimal debt contract and bank-run conditions match the description in Proposition 2. The regulator also internalizes how the volume of complex banks affects the endogenous determination of the complex-asset price in interbank markets, which means that the complex-asset price in bad times $P_C^W(b)$ is related to the volume of complex assets in a manner analogous to equation (1):

$$V^W R(1-L)\mu_b = \left(1 - V^W\right)\mu_b \frac{L - \kappa\phi}{P_C^W(b)}. \tag{6}$$

The welfare in the economy can then be written as

$$\mathcal{W}\left(V^W\right) = V^W \mathbb{E}\left[U_C | P_C(b) = P_C^W(b)\right] + \left(1 - V^W\right)\mathbb{E}\left[U_S | P_C(b) = P_C^W(b)\right]. \tag{7}$$

The equilibrium may exhibit excessive or insufficient investment in complex assets relative to the regulator's solution, depending on the magnitude of liquidity requirements relative to a threshold level.

Proposition 4 (Welfare-maximizing volume of complex banks)**.** *There exists $\hat{L}$ such that the following hold. When liquidity requirements are tight, $L > \hat{L}$, then there is excess investment in complex assets, i.e., $V^W < V^*$. Moreover, the welfare-maximizing complex-asset price in bad times is equal to the maximum level of 1, i.e., $P_C^W(b) = 1 > P_C^*(b)$. When liquidity requirements are loose, $L < \hat{L}$, then there is underinvestment in complex assets, i.e., $V^W > V^*$. Moreover, the welfare-maximizing complex-asset price in bad times $P_C^W(b)$ satisfies $0 < P_C^W(b) < P_C^*(b)$.*

The equilibrium is generically inefficient because the interbank market provides incomplete insurance. Banks do not take into account the impact of their portfolio choice on the interbank complex-asset price, and how that affects the quality of insurance that can be achieved on the interbank market.[17]

The intuition for the pivotal role of the tightness of liquidity requirements is as follows. On the one hand, the planner may have a stronger incentive to invest in complex assets compared to the individual banks because it internalizes the full return of these assets in the bad state, whereas the individual banks that invest in complex assets receive only a fraction of this return based on the interbank market price. On the other hand, the planner may have a stronger incentive to invest in simple assets because it internalizes that this would effectively distribute more liquidity to the liquidity-shocked depositors of the distressed banks. If liquidity requirements are sufficiently tight, each safe bank has a large amount of excess liquidity that can be used to buy assets from the distressed banks, so the latter effect dominates. Otherwise, the former effect dominates.[18]

4.4 Optimal Liquidity Requirements

We next discuss the optimal level of liquidity requirements. In particular, we show that it is optimal to keep liquidity requirements relatively loose regardless of whether a

[17] This is similar to the result in Geanakoplos and Polemarchakis (1985), which states that in the presence of incomplete markets, a competitive equilibrium is generically constrained inefficient.

[18] An alternative explanation that is slightly more mathematical is the following. The regulator's incentive to increase the complex-asset price is increasing in the level of the equilibrium complex-asset price. This is because increasing the price improves the performance of complex banks, which sell assets, but decreases the performance of simple banks that draw a high return, which buy assets. The marginal benefit of increasing the price is constant since it enters linearly into the complex-bank return (see equation (2)), whereas the marginal cost is decreasing in the level of the price since it enters hyperbolically into the simple-bank return (see equation (3)). Hence, the marginal net benefit of increasing the price is increasing in the level of the price. Now, consider a case in which the equilibrium complex-asset price in bad times $P_C^*(b)$ is increasing in the liquidity ratio L (see Proposition 3). If liquidity requirements exceed $\hat{L}$, then the equilibrium complex-asset price is high enough that the marginal net benefit of further increasing the complex-asset price is positive. In that case, the planner has an incentive to increase the complex-asset price relative to the equilibrium by reducing the volume of complex banks. An analogous argument holds if the liquidity level is lower than $\hat{L}$.

policymaker can simultaneously implement the efficient level of investment in complex assets.

Proposition 5 (Welfare-maximizing tightness of liquidity requirements)**.** *If a policymaker can implement the efficient level of investment in complex assets, then the optimal tightness of liquidity requirements is no greater than $\hat{L}$. If a policymaker allows the volume of complex assets to be determined in equilibrium, then the optimal tightness of liquidity requirements is also no greater than $\hat{L}$.*

Liquidity requirements have the benefit of improving a bank's liquidation value upon facing a run, but they also have an opportunity cost associated with curtailing investment in higher-yielding illiquid assets. Liquidity requirements are more likely to have a net cost when the complex-asset price in bad times is high. In particular, as the complex-asset price in bad times increases, complex banks facing a run are relatively better able to generate liquidity by selling their long-term assets compared to holding liquid assets. Additionally, simple banks are relatively less able to generate a return from investing their excess liquidity in the interbank market. In the case where a policymaker can implement the efficient level of investment in complex assets, the complex-asset price in bad times is high when $L > \hat{L}$, as shown in Proposition 4. In the case where a policymaker allows the volume of complex assets to be determined in equilibrium, liquidity requirements can also affect welfare by varying the discrepancy between the equilibrium and optimal prices. However, we show in the proof that this effect is insufficient to overcome the direct opportunity cost as L surpasses $\hat{L}$.[19]

5 Financial-stability Implications of Different Policies

The previous analysis has uncovered the conditions under which tighter liquidity requirements give rise to greater investment in complex assets, with potential repercussions for financial stability. This section describes how liquidity regulation interacts with three alternative policies aimed at fostering financial stability: unconventional monetary policy in the form of quantitative easing, an ex-ante insurance system, and asset-specific taxes.

[19]In simulations, we find that the optimal level of liquidity requirements L that maximizes welfare $\mathcal{W}$ is given by the minimum level consistent with the parametric assumptions in Propositions 1 and 2, regardless of whether the fraction of complex assets is determined in equilibrium or chosen by the planner (for example, see the bottom row of Figure E.1 in Online Appendix E). See Online Appendix C for a more general discussion of the comparative statics of the optimal L when it is equal to the minimum level that is consistent with the parametric assumptions in Propositions 1 and 2.

5.1 Quantitative Easing

By affecting interbank trading, our model naturally connects with the transmission of monetary policy through banks' funding costs, which are (at least partially) determined on the interbank market, and through the extent to which they are financially constrained, which is reflected by the liquidity composition of their asset side. Since the Great Financial Crisis, central banks around the world have responded by implementing unconventional monetary policies. In particular, quantitative easing (QE) refers to asset purchases by central banks. QE has been implemented by the Federal Reserve in the U.S. during both the Great Financial Crisis and the COVID-19 crisis to stabilize asset prices. This section analyzes how QE interacts with liquidity regulation, and describes conditions under which it may or may not improve welfare.

QE general implementation. To characterize the implementation of QE, we first enrich the model. We assume that at the beginning of period 1 each depositor randomly receives an income shock $\hat{\nu} \in \{0, \nu\}$, where it is commonly known that the probability of receiving ν is equal to δ. We assume δ is sufficiently small, so that all of the previous results still hold. After potentially paying an income tax, the depositors deposit their income in banks.

The government only charges a tax if the aggregate state is bad. Specifically, the government requires all depositors with a positive income shock to pay a tax τ, which creates a total tax revenue of $\tau\delta$. The government then uses the tax income to buy complex assets from the distressed banks. If the volume of complex banks at tax level τ is equal to $V(\tau)$, then the resulting equilibrium complex-asset price $P_C^\tau(b)$ satisfies

$$\underbrace{V(\tau) R\big(1-L\big)\mu_b}_{\text{complex-asset supply}} = \underbrace{\Big(1 - V(\tau)\Big)\,\mu_b\,\frac{L-\kappa\phi}{P_C^\tau(b)}}_{\text{complex-asset demand from simple banks}} + \underbrace{\frac{\tau\delta}{P_C^\tau(b)}}_{\text{complex-asset purchase by gov.}}. \tag{8}$$

Finally, in period 2 the government returns these assets to the late depositors as a lump sum. Given the debt contract and bank-run conditions in Proposition 2, the expected utility from investing in complex assets as a function of the complex-asset price in bad times $P_C(b)$ can now be written as

$$\begin{aligned}\mathbb{E}[U_C|P_C(b)] = \eta & \left(\underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + \mu_g R(1-L)}_{\text{return to normal dep.}} + \underbrace{\delta v}_{\text{income}} \right) \\ & + (1-\eta) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \left(\underbrace{L + P_C(b)\ \mu_b\ R(1-L) + \delta(\nu - \tau)}_{\text{liquidation value}} \right) + \underbrace{(1-\eta)\frac{\tau\delta}{P_C^{\tau}(b)}}_{\text{gov. payoff}},\end{aligned} \tag{9}$$

and the expected utility from investing in simple assets can now be written as

$$\begin{aligned}\mathbb{E}[U_S|P_C(b)] = \eta & \left(\mu_g \left(\underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + R(1-L)}_{\text{return to normal dep.}} + \underbrace{\delta\nu}_{\text{income}} \right) \right. \\ & \left. + (1-\mu_g) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \underbrace{(L + \delta\nu)}_{\text{liquidation value}} \right) \\ & + (1-\eta) \left(\mu_b \left(\underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{\frac{\overbrace{L - \kappa\phi}^{\text{buy complex assets}}}{P_C(b)} + R(1-L)}_{\text{return to normal dep.}} + \underbrace{\delta(\nu - \tau)}_{\text{income}} \right) \right. \\ & \left. + (1-\mu_b) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \underbrace{(L + \delta(\nu - \tau))}_{\text{liquidation value}} \right) + \underbrace{(1-\eta)\frac{\tau\delta}{P_C^{\tau}(b)}}_{\text{gov. payoff}}.\end{aligned} \tag{10}$$

We distinguish the welfare implications of QE based on whether it is anticipated or unanticipated.

Unanticipated QE. If QE is unanticipated, or implemented in a manner that comes as a surprise in period 1 after bank portfolios have already been determined, the government takes as given the volume of complex banks that would occur if banks expected no tax, $V(0)$, and chooses the tax τ to maximize welfare:

$$\mathcal{W}(\tau) = V(0)\mathbb{E}\left[U_C|P_C(b) = P_C^{\tau}(b)\right] + (1 - V(0))\,\mathbb{E}\left[U_S|P_C(b) = P_C^{\tau}(b)\right]. \tag{11}$$

Charging a higher tax rate allows the government to accrue more funds that it can use to buy complex assets, which in turn increases the complex-asset price.

Proposition 6. *If QE is unanticipated and $P_C^\tau(b) < 1$, then the equilibrium complex-asset price is increasing in the tax τ: $\frac{\partial P_C^\tau(b)}{\partial \tau} > 0$.*

Increasing the complex-asset price has the benefit of mitigating the severity of runs on complex banks during bad times, which always outweighs the cost of the tax when QE is unanticipated.[20]

Proposition 7 (Unanticipated QE)**.** *If QE is unanticipated, then the optimal tax is positive and equal to the minimum of income ν and the minimum tax necessary to increase the complex-asset price in bad times $P_C^\tau(b)$ to 1.*

Anticipated QE. If QE is anticipated, or implemented in a manner that can be predicted when banks choose their portfolios in period 0, the government internalizes the fact that increasing the price of complex assets in bad times affects the volume of banks that invest in complex assets, $V(\tau)$. Welfare then becomes

$$\mathcal{W}(\tau) = V(\tau)\mathbb{E}\left[U_C | P_C(b) = P_C^\tau(b)\right] + (1 - V(\tau))\,\mathbb{E}\left[U_S | P_C(b) = P_C^\tau(b)\right]. \tag{12}$$

Anticipated QE increases both the complex-asset price and the volume of complex banks.

Proposition 8. *If QE is anticipated and $P_C^\tau(b) < 1$, then*

(a) the equilibrium complex-asset price is increasing in the tax τ: $\frac{\partial P_C^\tau(b)}{\partial \tau} > 0$, and

(b) the equilibrium volume of complex banks is increasing in the tax τ: $\frac{\partial V(\tau)}{\partial \tau} > 0$.

Part (a) has a similar intuition as Proposition 6. Part (b) follows from the fact that increasing the complex-asset price partially insures against the runs experienced by complex banks in bad times, which strengthens the incentive for banks to invest in complex assets ex ante.[21]

This, in turn, has an offsetting negative effect on the complex-asset price. Therefore, when QE is anticipated, the attenuated benefit of increasing the complex-asset price in bad times can be smaller than the cost associated with the tax.

Proposition 9 (Anticipated QE)**.** *If QE is anticipated, the optimal tax can in general be either positive or zero. If the liquidity level L is sufficiently high, then the optimal tax is zero.*

[20] Consistent with these results, the right column of Figure E.2 in Online Appendix E shows that unanticipated QE is always associated with an increase in the complex-asset price and welfare.

[21] Recall from Table 1 that a disadvantage of complex assets relative to simple assets is incurring a run in the bad state, even if the asset would have yielded a high return.

The intuition is that anticipated QE has a weaker effect on the complex-asset price than unanticipated QE since it encourages more banks to invest in complex assets. When liquidity requirements are tight, then this shift towards complex assets reduces welfare since there is overinvestment in complex assets in equilibrium (see Proposition 4).[22] For sufficiently tight liquidity requirements, the benefit of QE increasing the complex-asset price falls short of the financing costs.[23]

Even though liquidity requirements dampen the effect of anticipated QE on welfare, in simulations we find that they amplify the effect on asset prices (see the top right panel of Figure E.3 in Online Appendix E). This is because they attenuate the complementarity between QE and investment in complex assets. In particular, they reduce the amount of complex assets an individual bank can hold, which reduces the benefit of the anticipated price support associated with QE. The weaker degree of substitution to complex assets therefore results in a weaker corresponding price reduction. Consistent with this, Online Appendix G.3 presents evidence that asset prices were more responsive to QE announcements after the implementation of the LCR compared to before.

5.2 Ex-ante Insurance

We next turn to an ex-ante insurance policy that always improves welfare. Consider the original environment as introduced in Section 4. If in period 1 the state is good, then in period 2 the government taxes high-return banks at the rate τ and distributes the proceeds equally to low-return banks. The tax is predictable in period 0. The tax rate is $\tau = 1 - \mu_g$, which sets equal the after-tax long-term return in good times for all banks:

$$\underbrace{(1-\tau)R(1-L)}_{\text{return of a high return bank}} = \underbrace{\tau\frac{\mu_g}{1-\mu_g}R(1-L)}_{\text{return of a low return bank}} = \mu_g R(1-L). \tag{13}$$

Note that the government must implement this arrangement since banks with a high realized return would have no incentive to honor a promise to pay the banks with a low realized return. This policy always improves welfare.

Proposition 10 (Ex-ante insurance)**.** *Implementing the ex-ante insurance policy (i) increases the equilibrium complex-asset price in bad times, (ii) decreases the volume of complex banks, and*

[22]Note that when liquidity requirements are loose, this shift towards complex assets improves welfare since there is underinvestment in complex assets in equilibrium.

[23]In line with this result, the bottom right panel of Figure E.3 in Online Appendix E shows that tightening liquidity requirements decreases the effect of QE on welfare, as banks are less reliant on asset prices as a means to respond to liquidity stress.

(iii) increases overall welfare.

The intuition is as follows. The ex-ante insurance policy increases the period-2 income of simple banks with a low return such that they no longer experience a run. This directly increases the expected utility from investing in simple assets since by avoiding runs in the good state, it shifts a greater share of the expected return of a simple bank to liquidity-shocked depositors with a higher marginal utility.[24]

Since the policy is predictable, it additionally motivates banks to switch to simple assets ex ante, which allows a greater fraction of banks to benefit from the redistribution in the good state.[25] This shift away from complex assets, in turn, leads to a reduction in liquidity demand during bad times and, thus, an increase in the equilibrium complex-asset price.[26]

Note that there is no incentive to implement an analogous redistribution in the bad state because the average return is less than the promised repayment to the early depositors due to the first inequality in (15). In particular, committing to redistribute in the bad state would trigger a run on all banks.

5.3 Implementation through Asset-specific Taxes

Finally, we consider asset-specific taxes as a means of implementing the constrained-efficient volume of complex banks (similar to Dávila and Korinek, 2018). For this purpose, recall that the equilibrium is generically inefficient and that the degree of investment in complex assets can be greater or less than in the planner solution depending on whether liquidity requirements are tighter or looser than a threshold level $\hat{L}$, respectively (Proposition 4). This subsection first shows that QE and ex-ante insurance cannot always be used to implement the constrained-efficient volume of complex banks. It then provides conditions under which the constrained-efficient volume of complex banks can be implemented with a tax on either complex or simple assets.

Proposition 11. *If $L < \hat{L}$ and v is sufficiently large, then the constrained-efficient volume of complex banks can be implemented via anticipated QE. However, the tax that implements the*

[24] Recall from Table 1 that a disadvantage of simple assets relative to complex assets is the risk of a run in the good state.

[25] Note that as more banks switch to simple assets, the price for complex assets in bad times increases, which decreases the expected utility of simple banks relative to the case where there is no adjustment of bank portfolios. However, the net effect of the adjustment on the expected utility across all banks is clearly positive because banks only switch to simple assets when the expected utility is greater compared to sticking with complex assets.

[26] The three results in Proposition 10 are illustrated in the right column of Figure E.4 in Online Appendix E.

constrained-efficient volume of complex banks may not be welfare-optimizing. If $L > \hat{L}$, then neither QE nor the ex-ante insurance policy can implement the constrained-efficient volume of complex banks.

The intuition is as follows. If $L < \hat{L}$, then the constrained-efficient volume of complex banks is greater than under the equilibrium solution (Proposition 4). Recall that anticipated QE increases the incentive to invest in complex assets since it supports the complex-asset price in bad times (Proposition 8). In particular, there is a tax that implements the constrained-efficient volume of complex banks. However, this tax level does not necessarily maximize welfare because QE also affects welfare through channels other than the volume of complex banks. To illustrate this, Figure 1 shows how the volume of complex banks and total welfare vary with the tax τ used to implement anticipated QE.

If $L > \hat{L}$, then the constrained-efficient volume of complex banks is less than that in the equilibrium solution (Proposition 4). The constrained-efficient volume of complex banks cannot be implemented with QE, since unanticipated QE has no effect on the volume of complex banks and anticipated QE can only increase the volume of complex banks. The ex-ante insurance policy decreases the volume of complex banks (Proposition 10), but not enough to implement the constrained-efficient level.

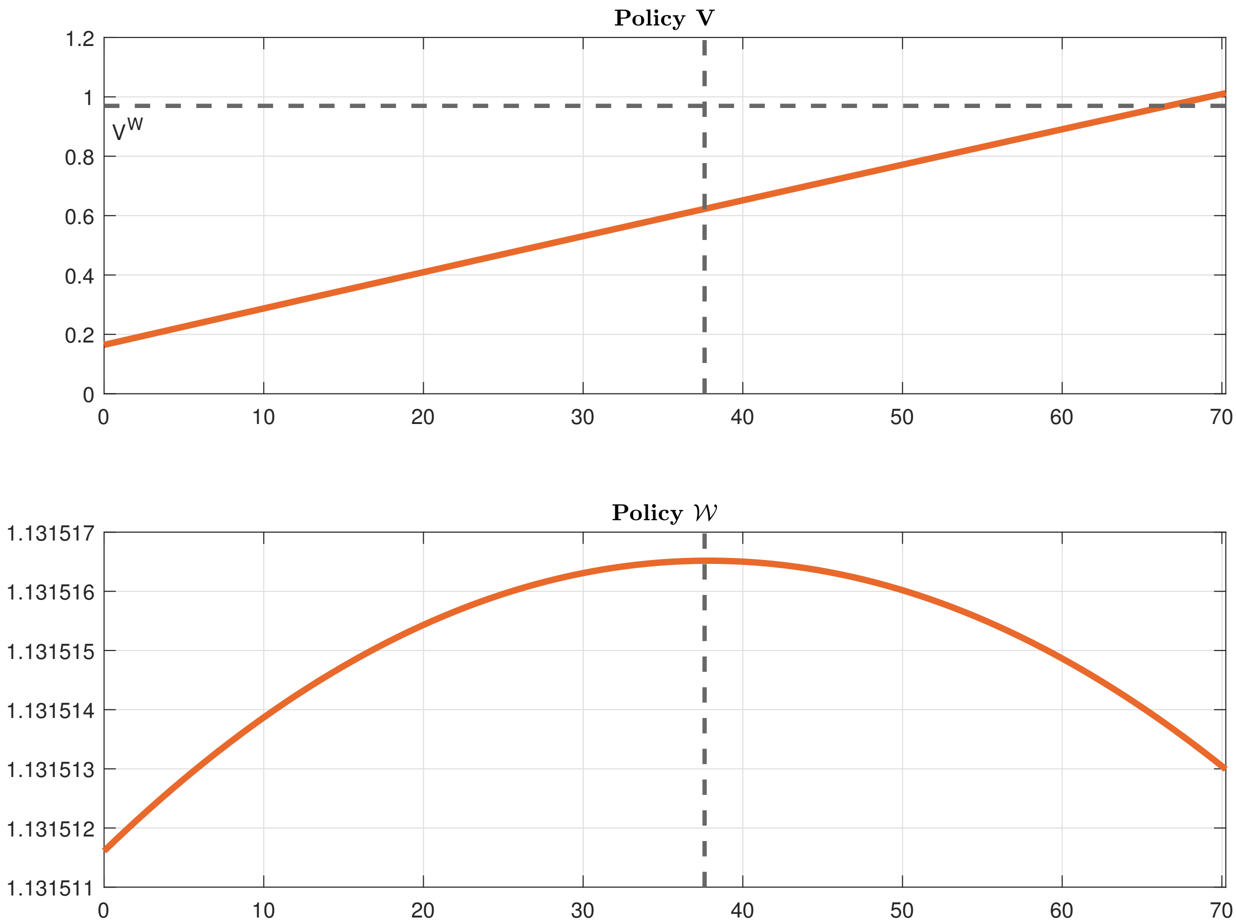
The constrained-efficient volume of complex banks for any level of L can be implemented with a tax on either complex or simple assets. The tax can be described as follows: if in period 1 the state is good, then in period 2 the government taxes high-return complex (or simple) banks at a rate of τ, and distributes the proceeds equally to the high-return simple (or complex) banks.

Proposition 12. *Denote by $\Delta(P_C(b)) = \mathbb{E}[U_C|P_C(b)] - \mathbb{E}[U_S|P_C(b)]$ the relative benefit of investing in complex assets without the tax as expressed in equation (4), by V^W the constrained-efficient volume of complex banks, and by $P_C^W(b)$ the complex-asset price in bad times for the constrained-efficient allocation. Then the following hold:*

- *If $L < \hat{L}$ and $\frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)} < \frac{R(1-L)-(\kappa-L)}{R(1-L)}$, then the constrained-efficient volume of complex banks can be implemented by transferring from simple to complex banks via a tax at the rate $\tau^* = \frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)}$.*
- *If $L > \hat{L}$ and $\frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)} < \frac{\mu_g R(1-L)-(L-\kappa)}{\mu_g R(1-L)}$, then the constrained-efficient volume of complex banks can be implemented by transferring from complex to simple banks via a tax at the rate $\tau^* = \frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)}$.*

Additionally, the tax level that implements the constrained-efficient volume of complex banks also maximizes welfare.

Figure 1: Variation in τ under anticipated QE. This figure shows how the volume of complex banks and welfare vary with τ under anticipated QE. The vertical dashed line corresponds to the welfare-optimizing tax. The horizontal dashed line corresponds to the constrained-efficient volume of complex banks. The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: $R = 1.104$, $L = 0.179$, $\kappa = 1.018$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.



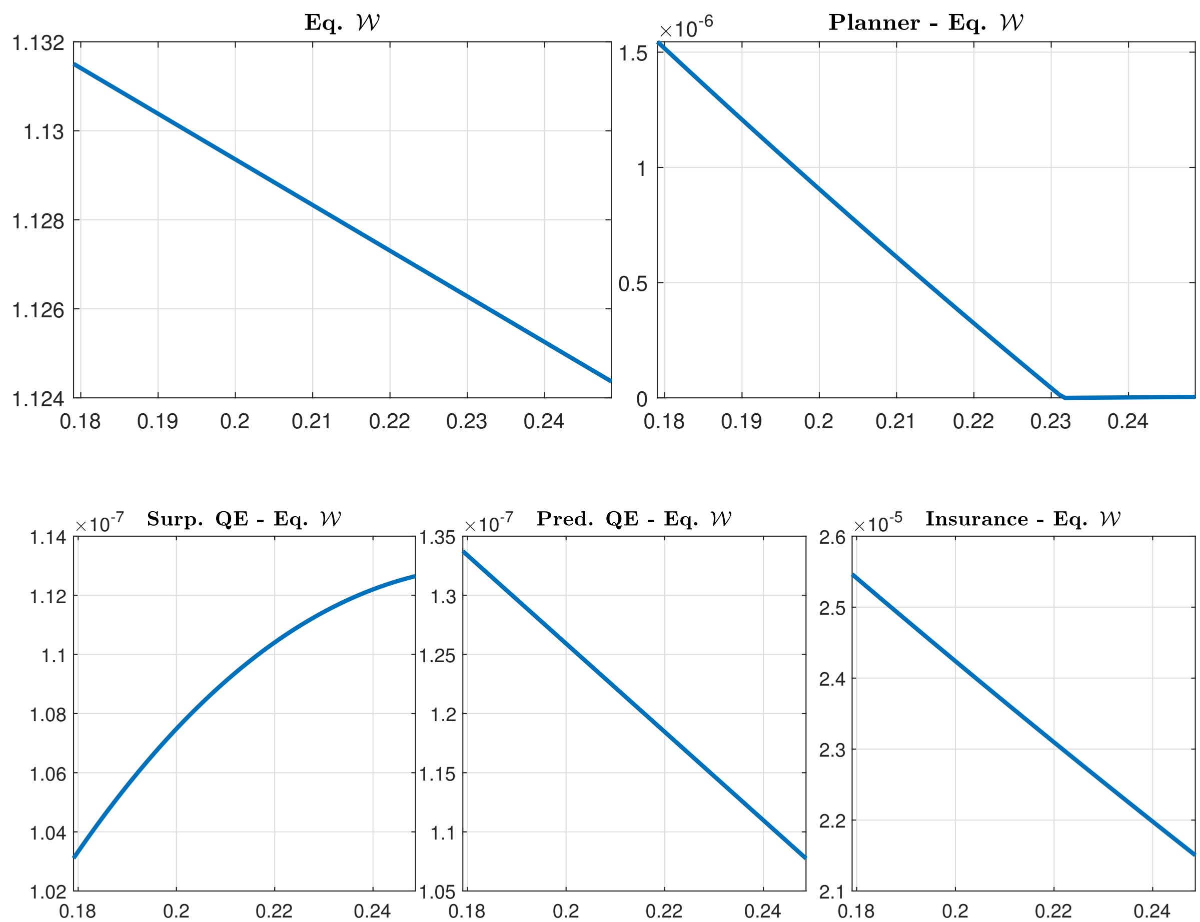
Note that the parametric assumptions in this proposition ensure that the tax is consistent with the incentive-compatibility conditions supporting an equilibrium of the form as described in Proposition 2.

5.4 Comparison of Policies

Figure 2 compares welfare as a function of the liquidity level L based on simulations of the various policy scenarios. The first row shows welfare in the baseline equilibrium in the version of the model with income shocks, as introduced at the beginning of Section 5.1. It also shows the improvement in utility associated with the constrained-efficient volume of complex banks, which can be implemented using asset-specific taxes (see Proposition 12).

The second row shows the welfare gains associated with unanticipated QE, anticipated QE, and ex-ante insurance. At a parameter set motivated by the Great Financial Crisis (as described in Online Appendix D), the ex-ante insurance policy achieves the greatest welfare gain across various levels of liquidity requirements. If liquidity requirements are loose, then it is followed by the planner solution, anticipated QE, and unanticipated QE. For tighter liquidity requirements, unanticipated QE may become more effective than the planner solution and anticipated QE.

Figure 2: Comparison of welfare gains under different policies. This figure shows welfare as a function of the liquidity level L in the baseline equilibrium with income shocks as well as the improvement in utility associated with unanticipated QE ("Surp. QE"), anticipated QE ("Pred. QE"), and the ex-ante insurance policy. The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: $R = 1.104$, $L = 0.179$, $\kappa = 1.018$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.



6 Conclusion

The Liquidity Coverage Ratio, alongside the Net Stable Funding Ratio, has been put in place to foster financial stability by forcing large banks to maintain sufficient liquidity on their balance sheets. This paper shows under what conditions tighter liquidity requirements substitute for or complement banks' investment in complex assets, such as structured securities, that may contribute to destabilizing trends in the economy.

In our model, the symmetric opacity associated with complex assets supports bank liquidity in good times, but it has a mixed effect on liquidity during crises. On the one hand, it causes panic-stricken depositors to run on banks that may turn out to be solvent. On the other hand, it also allows banks to draw liquidity from interbank markets. The model shows that tighter liquidity requirements can support asset prices during crises by increasing the supply of liquidity in interbank markets, but by doing so, it can also encourage greater investment in complex assets beforehand.

We provide a rich assessment of the welfare properties of the interaction of liquidity regulation and other policies aimed at fostering financial stability. First, the degree of investment in complex assets can be inefficiently high or low depending on liquidity requirements. Therefore, the tightness of liquidity requirements determines the asset-specific taxes that can be used to implement the constrained-efficient investment in complex assets. Second, liquidity regulation can undermine the benefit of ex-post interventions such as unconventional monetary policy, in particular quantitative easing (QE). This is more likely to be true if QE is implemented in a predictable manner, in which case the benefit of QE in supporting asset prices is offset by higher ex-ante investment in complex assets.

Overall, these results highlight several ways in which banks' endogenous investment in complex assets can influence the effects of liquidity requirements as well as their interactions with central-bank responses to the Great Financial Crisis and other financial-stability policies.

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Proofs

Proof of Proposition 1

Proposition 1 (Simplified equilibrium)**.** *There exists an equilibrium in which the following hold:*

1. *Banks pay depositors that withdraw early a return of $R_d = \kappa$.*
2. *Liquidity-shocked depositors always withdraw early, and normal depositors withdraw early if and only if*
   - *the bank is complex and the economic state is bad.*
   - *the bank is simple and its individual return is low.*
3. *The price for simple assets is $P_S^*(\omega) = 1$, the complex-asset price in good times is $P_C^*(g) = 1$, and, for V sufficiently large, the complex-asset price in bad times related to the volume of complex banks by the corresponding interbank-market clearing condition:*

$$\underbrace{VR(1-L)\mu_b}_{\text{complex-asset supply}} = \underbrace{\Big(1-V\Big)\ \mu_b\, \frac{L-\kappa\phi}{P_C^*(b)}}_{\text{complex-asset demand}} \tag{1}$$

4. *For suitable levels of V, the complex-asset price in bad times satisfies $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$.*

We prove each part of this proposition in turn, taking the other parts as given. Note that we will assume that V is within the range where $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$ holds.

Lemma 1 (Debt contract)**.** *Suppose the following conditions hold:*

$$R < \min\left(\frac{L(1-\phi)}{1-L}(\alpha-1), \frac{L(1-\phi)}{1-L\phi}\alpha\right) \tag{14}$$

$$(L+\mu_b R(1-L))\left(1+\frac{(1-\eta)(1-\phi)}{\eta\phi}\right) < \kappa < L + \mu_g R(1-L) \tag{15}$$

Then banks pay depositors that withdraw early a return of $R_d = \kappa$.

Proof. Before going into the proof, we first explain the intuition behind the parametric restrictions. The restriction in (14) ensures that the urgency for liquidity α for the liquidity-shocked depositors is large enough relative to the maximal return on long-term assets R, so that it is incentive compatible for liquidity-shocked depositors to always withdraw early. The restriction in (15) ensures that the expected return for complex banks,

$L + \mu_\omega R(1-L)$, exhibits a large enough disparity between the bad state and the good state, so that it is incentive compatible for normal depositors to withdraw under the described conditions. The first inequality in (15) also ensures that the liquidity need κ is large enough that banks, when deciding the debt contract R_d, are willing to experience a run in bad times to meet the full need in good times.

As for the proof itself, first observe that since liquidity-shocked depositors only have elevated marginal utility for the first κ of payments, it is clear that a bank has no incentive to pay more than κ.[27] Next, we show that it is also not optimal for banks to offer a rate lower than κ. For the rest of the proof, assume $R_d \le \kappa$. Note that the assumption $L > \kappa\phi$ implies that, if $R_d \le \kappa$ and only the mass ϕ of liquidity-shocked depositors withdraws early, then a bank has excess liquidity $L - R_d\phi > 0$, which can be used to buy up to $(L - R_d\phi)/P_C^*(\omega)$ normalized units of complex assets.

Complex banks. Consider a complex bank. We consider three cases depending on the magnitude of the early repayment R_d.

Case 1: If $L + P_C^*(b)\mu_b R(1-L) < R_d \le \kappa$, we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors withdraw early only in the bad state. Note that this case is well-defined since $L + P_C^*(b)\mu_b R(1-L) < \kappa$ follows from the assumption in the first inequality in (15).

If the economic state in period 1 is good, a mass ϕ of liquidity-shocked depositors withdraws in period 1, and a mass $1-\phi$ of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is $\frac{L - R_d\phi + \mu_g R(1-L)}{1-\phi}$. Therefore the best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} &\frac{L - R_d\phi + \mu_g R(1-L)}{1-\phi} > R_d \\ &\iff L + \mu_g R(1-L) > R_d. \qquad (16) \end{aligned}$$

This is satisfied since $R_d \le \kappa < L + \mu_g R(1-L)$ by the second inequality in (15). The best response for an individual liquidity-shocked depositor is to withdraw early if and only

[27] Note that an individual bank is indifferent to paying slightly higher amounts than κ as long as normal depositors do not have an incentive to withdraw early. However, we rule out these equilibria because they are socially less efficient due to their effects on the equilibrium asset price.

if

$$\frac{L - R_d\phi + \mu_g R(1-L)}{1-\phi} < \alpha R_d$$
$$\iff \frac{L + \mu_g R(1-L)}{\alpha(1-\phi)+\phi} < R_d. \tag{17}$$

This is satisfied since $R_d > L + P_C^*(b)\mu_b R(1-L) > \frac{L+\mu_g R(1-L)}{\alpha(1-\phi)+\phi}$ by (14) (when the minimum is the left term).

If the economic state in period 1 is bad and all depositors withdraw early, then the bank defaults since the maximum liquidity it can supply by paying out of its liquid assets, L, and by selling assets, $\mu_b P_C^*(b)R(1-L)$, is less than the demand, R_d. Therefore the best response for an individual normal depositor is to withdraw early since the payment from withdrawing early, which is the total liquidation value of the bank $L + P_C^*(b)\mu_b R(1-L)$ since the bank experiences a run, is greater than the payment from withdrawing late, which is zero.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\mathbb{E}[U_C] = \eta\left(\alpha R_d\phi + L - R_d\phi + \mu_g R(1-L)\right) + (1-\eta)\Big(\alpha\phi + 1 - \phi\Big)\Big(L + P_C^*(b)\mu_b R(1-L)\Big).$$

The locally optimal R_d is the upper bound κ since $\alpha > 1$. The maximum expected utility is then

$$\mathbb{E}[U_C | R_d = \kappa] = \eta\left(\alpha\kappa\phi + L - \kappa\phi + \mu_g R(1-L)\right) + (1-\eta)\Big(\alpha\phi + 1 - \phi\Big)\Big(L + P_C^*(b)\mu_b R(1-L)\Big). \tag{18}$$

Case 2: If $\max\left\{\frac{L/P_C^*(\omega)+\mu_\omega R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}\right\}_{\omega=b,g} \le R_d \le L + P_C^*(b)\mu_b(1-L)R$, we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.[28] Note that this case is well-defined since $\frac{L+\mu_g R(1-L)}{\alpha(1-\phi)+\phi} \le L + P_C^*(b)\mu_b(1-L)R$ follows from the assumption in (14) (when the minimum is the left term), and $\frac{L/P_C^*(b)+\mu_b R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(b)} < L + P_C^*(b)\mu_b(1-L)R$ follows from $\alpha P_C^*(b) > 1$.

[28]Note that there is no equilibrium in which normal depositors withdraw early. To see this, note that the bank cannot default in period 1 even if all depositors withdraw early. If all other depositors withdraw early, then the best response for an individual normal depositor is to withdraw late because the individual payoff is infinite.

If the economic state in period 1 is good and the equilibrium is as described, then the best response for an individual normal depositor is to withdraw late since $R_d \le \kappa$ implies that the condition in equation (16) is satisfied. The best response for an individual liquidity-shocked depositor is to withdraw early since $R_d > \frac{L+\mu_g R(1-L)}{\alpha(1-\phi)+\phi}$ implies that the condition in equation (20) is satisfied.

If the economic state in period 1 is bad, a mass ϕ of liquidity-shocked depositors withdraws in period 1, and a mass $1-\phi$ of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is $\frac{(L-R_d\phi)/P_C^*(b)+\mu_b R(1-L)}{1-\phi}$. The best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} &\frac{(L-R_d\phi)/P_C^*(b)+\mu_b R(1-L)}{1-\phi} > R_d \\ \iff &\frac{L/P_C^*(b)+\mu_b R(1-L)}{1-\phi+\phi/P_C^*(b)} > R_d. \end{aligned} \tag{19}$$

This is satisfied since $R_d \le L+\mu_b P_C^*(b)(1-L)R < \frac{L/P_C^*(b)+\mu_b R(1-L)}{1-\phi+\phi/P_C^*(b)}$, which follows from $P_C^*(b) < 1$. The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} &\frac{(L-R_d\phi)/P_C^*(b)+\mu_b R(1-L)}{1-\phi} \le \alpha R_d \\ \iff &\frac{L/P_C^*(b)+\mu_b R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(b)} \le R_d. \end{aligned} \tag{20}$$

This is satisfied since $R_d \ge \frac{L/P_C^*(b)+\mu_b R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(b)}$.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\begin{aligned} \mathbb{E}[U_C] = &\,\eta\big(\alpha R_d\phi + L - R_d\phi + \mu_g R(1-L)\big) \\ &+ (1-\eta)\left(\alpha R_d\phi + \frac{L-R_d\phi}{P_C^*(b)} + \mu_b R(1-L)\right). \end{aligned} \tag{21}$$

The locally optimal R_d is the upper bound $L+\mu_b P_C^*(b)(1-L)R$ since $\alpha > 1$ and $\alpha P_C^*(b) >$

1. The maximum expected utility can then be written as

$$\mathbb{E}\left[U_C | R_d = L + \mu_b P_C^*(b)(1-L)R\right] = \eta\left(\phi(\alpha-1)\left[L + P_C^*(b)\mu_b(1-L)R\right] + L + \mu_g R(1-L)\right) + (1-\eta)\left[L + P_C^*(b)\mu_b(1-L)R\right]\left(\alpha\phi + \frac{1-\phi}{P_C^*(b)}\right). \quad (22)$$

Case 3: If $R_d < \max\left\{\frac{L/P_C^*(\omega)+\mu_\omega R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}\right\}_{\omega=b,g}$, then there is no equilibrium in which liquidity-shocked fdepositors withdraw early in both states since at least one of (17) or (20) is violated. If liquidity-shocked depositors withdraw late in the bad state, then the utility of the bank in the bad state, $\frac{L}{P_C^*(b)} + \mu_b R(1-L)$, is less than the utility in the bad state from Case 2 in equation (21) since $\alpha P_C^*(b) > 1$. Similarly, if liquidity-shocked depositors withdraw late in the good state then the utility of the bank in the good state, $L + \mu_g R(1-L)$, is less than the utility in the good state from Case 2 in equation (21) since $\alpha > 1$. Therefore Case 3 is dominated by Case 2.

To determine the globally optimal payment R_d, we compare the maximum utility between Case 1 and Case 2. The expected utility from Case 1 (equation (18)) minus the expected utility from Case 2 (equation (22)) is

$$\eta\phi(\alpha-1)\left(\kappa - \left[L + P_C^*(b)\mu_b R(1-L)\right]\right) - (1-\eta)(1-\phi)\left[L + P_C^*(b)\mu_b R(1-L)\right]\left(\frac{1}{P_C^*(b)} - 1\right) > 0, \quad (23)$$

where the inequality follows by $1 > P_C^*(b) > \frac{1}{\alpha}$ and the assumption in the first inequality in (15).[29] This implies the globally optimal payment is $R_d = \kappa$.

Simple banks. Consider a simple bank. We consider three cases depending on the magnitude of the early repayment R_d.

Case 1: If $L < R_d \leq \kappa$, we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors withdraw early only when the bank's return is low. Note that this case is well-defined since $L < \kappa$.

If return is high, the economic state is ω, a mass ϕ of liquidity-shocked depositors withdraws in period 1, and a mass $1-\phi$ of normal depositors withdraws in period 2,

[29] To see this, denote the left-hand side of the first inequality in (15) by F. Note that $L + P_C^*(b)\mu_b R(1-L) < L + \mu_b R(1-L)$ since $P_C^*(b) < 1$. Let $x \equiv L + \mu_b R(1-L)$. Therefore $F \geq \eta\phi(\alpha-1)(\kappa - x) - (1-\eta)(1-\phi)x\left(\frac{1}{P_C^*(b)} - 1\right)$. Then note that $\frac{1}{P_C^*(b)} < \alpha$, which implies $F \geq \eta\phi(\alpha-1)(\kappa-x) - (1-\eta)(1-\phi)x(\alpha-1)$. Then the result follows from rearranging and applying the first inequality in (15). Intuitively, a large enough liquidity shock κ ensures that banks are willing to experience a run in bad times in order to meet the full need in good times.

then the payment in period 2 for an individual normal depositor is $\frac{(L-R_d\phi)/P_C^*(\omega)+R(1-L)}{1-\phi}$. Therefore the best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} &\frac{(L-R_d\phi)/P_C^*(\omega)+R(1-L)}{1-\phi} > R_d \\ \iff &\frac{L/P_C^*(\omega)+R(1-L)}{1-\phi+\phi/P_C^*(\omega)} > R_d. \end{aligned} \tag{24}$$

This is satisfied since $R_d \leq \kappa < \frac{L/P_C^*(\omega)+R(1-L)}{1-\phi+\phi/P_C^*(\omega)}$.[30] The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} &\frac{(L-R_d\phi)/P_C^*(\omega)+R(1-L)}{1-\phi} < \alpha R_d \\ \iff &\frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)} < R_d. \end{aligned} \tag{25}$$

For $\omega = b$, this follows from $L < R_d$, $P_C^*(b) > \frac{1}{R}$ and the assumption in (14) (when the minimum is the right term), and for $\omega = g$ it follows from $L < R_d$, $P_C^*(g) = 1$ and the assumption in (14) (when the minimum is the left term).

If the return revealed in period 1 is low and all depositors withdraw early, then the bank defaults since the maximum liquidity it can supply by paying out of its liquid assets L is less than the demand R_d.[31] Therefore the best response for an individual normal depositor is to withdraw early since the payment from withdrawing early, which is the total liquidation value of the bank L since the bank experiences a run, is greater than the payment from withdrawing late, which is zero.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\begin{aligned} \mathbb{E}[U_S] = \eta\left(\mu_g\left(\alpha R_d\phi + L - R_d\phi + R(1-L)\right) + (1-\mu_g)(\alpha\phi+1-\phi)L\right) \\ + (1-\eta)\left(\mu_b\left(\alpha R_d\phi + \frac{L-R_d\phi}{P_C^*(b)} + R(1-L)\right) + (1-\mu_b)(\alpha\phi+1-\phi)L\right). \end{aligned}$$

The locally optimal R_d is the upper bound κ since $\alpha > 1$ and $\alpha P_C^*(b) > 1$.

[30]To see this, note that $\frac{L/P_C^*(\omega)+R(1-L)}{1-\phi+\phi/P_C^*(\omega)}$ is between $L + R(1-L)$ (when evaluated at $P_C^*(\omega) = 1$) and $\frac{L}{\phi}$ (when evaluated at the limit as $P_C^*(\omega) \to 0$). Then note that $\kappa < L + R(1-L)$ by the second inequality in (15) and $\kappa < \frac{L}{\phi}$ by the assumption $L > \kappa\phi$.

[31]Recall that the bank cannot sell its observably worthless assets.

Case 2: If $\max\left\{\frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}\right\}_{\omega=b,g} \leq R_d \leq L$, we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.[32] Note that this case is well-defined since $\frac{L/P_C^*(b)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(b)} < L$ follows from $P_C^*(b) > \frac{1}{R}$ and the assumption in (14) (when the minimum is the right term) while $\frac{L/P_C^*(g)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(g)} < L$ follows from $P_C^*(g) = 1$ and the assumption in (14) (when the minimum is the left term).

If the individual return in period 1 is high and the equilibrium is as described, then the best response for an individual normal depositor is to withdraw late since $R_d \leq \kappa$ implies that the condition in (24) is satisfied. The best response for an individual liquidity-shocked depositor is to withdraw early since $R_d \geq \frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}$ implies that the condition in (25) is satisfied.

If the return in period 1 is low, the economic state is ω, a mass ϕ of liquidity-shocked depositors withdraws in period 1, and a mass $1-\phi$ of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is $\frac{(L-R_d\phi)/P_C^*(\omega)}{1-\phi}$. The best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} &\frac{(L-R_d\phi)/P_C^*(\omega)}{1-\phi} > R_d \\ \iff &\frac{L/P_C^*(\omega)}{1-\phi+\phi/P_C^*(\omega)} \geq R_d. \end{aligned} \tag{26}$$

This is satisfied since $P_C^*(\omega) \leq 1$ implies $\frac{L/P_C^*(\omega)}{1-\phi+\phi/P_C^*(\omega)} \geq L \geq R_d$. The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} &\frac{(L-R_d\phi)/P_C^*(\omega)}{1-\phi} \leq \alpha R_d \\ \iff &\frac{L/P_C^*(\omega)}{\alpha(1-\phi)+\phi/P_C^*(\omega)} \leq R_d. \end{aligned} \tag{27}$$

This is satisfied since $R_d \geq \frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}$.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of

[32]Note that there is no equilibrium in which normal depositors withdraw early when the return is low by a similar argument as in Case 2 for complex banks.

the bank is

$$\mathbb{E}[U_S] = \eta\left(\alpha R_d\phi + L - R_d\phi + \mu_g R(1-L)\right) \\ + (1-\eta)\left(\alpha R_d\phi + \frac{L - R_d\phi}{P_C^*(b)} + \mu_b R(1-L)\right). \tag{28}$$

The locally optimal R_d is the upper bound $R_d = L$ since $\alpha > 1$ and $\alpha > \frac{1}{P_C^*(b)}$. This portfolio is dominated by investing in complex investments and setting $R_d = L + P_C^*(b)\mu_g R(1-L)$. This can be seen by observing that the expected utility is the same function of R_d as Case 2 for a complex bank (equation (21)), this function is increasing in R_d, and the local optimum from Case 2 for a complex bank $R_d = L + P_C^*(b)\mu_b R(1-L)$ is larger than the local optimum for Case 2 of a simple bank L.

Case 3: If $R_d < \max\left\{\frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}\right\}_{\omega=b,g}$, then there is no equilibrium in which liquidity-shocked depositors withdraw early in both aggregate states since (25) fails for either $\omega = b$ or $\omega = g$. If the liquidity-shocked depositors withdraw late in the bad state, then the expected utility of the bank in the bad state, $\frac{L}{P_C^*(b)} + \mu_b R(1-L)$, is less than the utility in the low return state from Case 2 in equation (28) since $\alpha P_C^*(b) > 1$. Similarly, if the liquidity-shocked depositors withdraw early in the high return state then the expected utility of the bank in the high return state, $L + \mu_g R(1-L)$, is less than the utility in the good state from Case 2 in equation (28) since $\alpha > 1$. Therefore Case 3 is dominated by Case 2.

Therefore, if a bank invests in simple assets, then the optimal repayment R_d must correspond to the local maximum from Case 1, which is $R_d = \kappa$. □

Corollary 1 (Bank-run conditions)**.** *Liquidity-shocked depositors always withdraw early and normal depositors withdraw early if and only if*

- *the bank is complex and the economic state is bad.*
- *the bank is simple and the observable return is low.*

Proof. This follows from the proof of Lemma 1 for the special case where $R_d = \kappa$. □

Lemma 2 (Interbank market equilibrium)**.** *The price for simple assets is $P_S^*(\omega) = 1$, the complex-asset price in good times is $P_C^*(g) = 1$, and the complex-asset price in bad times is either $P_C^*(b) = 1$ or related to the volume of complex banks by the corresponding interbank-market* *clearing condition:*

$$\underbrace{VR(1-L)\mu_b}_{\text{complex-asset supply}} = \underbrace{\Big(1-V\Big)\ \mu_b\ \frac{L-\kappa\phi}{P_C^*(b)}}_{\text{complex-asset demand}} \tag{29}$$

Proof. Note that $P_S^*(\omega)=1$ since there is no market for simple assets, as simple banks with an observably low return cannot sell their assets while simple banks with an observably high return have no incentive to do so. Therefore, the price is at the maximum level of 1, which corresponds to the return on liquid assets. Similarly, $P_C^*(g)=1$ since there is no market for complex assets in the good state, as no complex banks experience a run in that state.

Now consider $P_C^*(b)$. In the bad state, all complex banks sell their assets. In particular, each complex bank sells $\mu_b(1-L)R$ normalized units of complex assets. Therefore, the overall supply of complex assets is

$$\int_{i:\xi_i=C} S_{B,i}(P_C^*(b))di = \int_{i:\xi_i=C} \mu_b(1-L)Rdi = \mu_b(1-L)R\ V. \tag{30}$$

At the same time, simple banks with a positive return have excess liquidity, which they fully invest in complex assets since $P_C^*(b)<1$. In particular, each simple bank with a positive return demands $\frac{L-\kappa\phi}{P_C^*(b)}$ normalized units of complex assets. Therefore, the overall demand for complex assets is

$$\int_{i:\xi_i=S} \frac{L-\kappa\phi}{P_C^*(b)}\ \mathbf{1}_{\{R_i^S(b)=R\}}\ di = \frac{L-\kappa\phi}{P_C^*(b)}\quad \underbrace{\mu_b\Big(1-V\Big)}_{\substack{\text{mass of simple banks}\\ \text{with postive return}}}\quad . \tag{31}$$

Equating market supply and market demand for complex assets (i.e., Eqs. (30) and (31)), implies the result. Note however that the price cannot be greater than 1, otherwise the simple banks would substitute to liquid assets for a better return. Instead, if the complex-asset supply is sufficiently low, (1) breaks as simple banks with a positive return only partially invest in complex assets. □

Corollary 2 (Complex-asset price (exogenous V))**.** *If*

$$\frac{L-\kappa\phi}{R(1-L)+L-\kappa\phi} < V < \frac{L-\kappa\phi}{1-\kappa\phi} \tag{32}$$

then the price for complex assets in the bad state satisfies $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$.

Proof. A direct application of Lemma 2 implies that $\frac{1}{R} < P_C^*(b) < 1$. Note also that $\frac{1}{\alpha} < \frac{1}{R}$ follows from the assumption in (14) (when the minimum is the right term). □

Proof of Proposition 2

Proposition 2 (Equilibrium)**.** *There exists an equilibrium in which:*

1. *Properties 1-3 of the equilibrium from Proposition 1 are satisfied.*
2. *All banks invest in long-term assets without any excess liquidity.*
3. *At the equilibrium volume of complex banks V^*, the complex-asset price in bad times satisfies $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$.*

We prove each part of this proposition in turn.

Lemma 3. *Properties 1-3 of the equilibrium from Proposition 1 are satisfied.*

Proof. Lemma 5 shows that both types of risky assets are held in equilibrium. Therefore, properties 1-3 of the equilibrium from Proposition 1 are satisfied at the equilibrium V^* since they are satisfied for any $V \in (0,1)$. □

Lemma 4. *If*

$$\frac{\eta(\alpha\phi + 1 - \phi) + (1-\eta)\alpha}{\eta\mu_g + (1-\eta)\mu_b} < R, \tag{33}$$

then all banks invest in long-term assets without any excess liquidity.

Proof. Before going into the proof, we first explain the intuition behind the parametric restriction. In particular, (33) implies that the return for the two types of long-term assets R is sufficiently high, so that banks do not have an incentive to hold excess liquidity.

The proof has two parts. In the first part, we show that holding only liquid assets is a dominated portfolio, even allowing the bank to simultaneously choose the debt contract R_d. In the second part, we show that banks have no incentive to hold excess liquidity conditional on investing in either complex or simple assets and maintaining a debt contract of $R_d = \kappa$.

Part 1: In Part 1, we show that holding only liquid assets is a dominated portfolio. To determine the optimal early repayment for a bank fully invested in liquid assets, we consider four cases depending on the magnitude of the early repayment R_d.

Case 1: If $R_d > \frac{1}{\phi+(1-\phi)P_C^*(b)}$, then the bank always experiences a run. To see this, first note that liquidity-shocked depositors always withdraw early since if all depositors withdraw late then the payoff is $\frac{1}{P_C^*(\omega)}$, which is less than α in equilibrium. However, if liquidity-shocked depositors withdraw early then they receive a payoff of $\alpha R_d > \frac{\alpha}{\phi+(1-\phi)P_C^*(\omega)} > \alpha$ if $R_d \leq \kappa$, or $\alpha\kappa + R_d - \kappa > \alpha$ if $R_d > \kappa$. Given that liquidity-shocked depositors withdraw early, normal depositors will also withdraw early since the payoff from withdrawing late is $\frac{(1-R_d\phi)/P_C^*(\omega)}{1-\phi} < \frac{1}{\phi+(1-\phi)P_C^*(b)} \frac{P_C^*(b)}{P_C^*(\omega)} \leq R_d$. Since the bank always experiences a run, the utility of the bank is

$$\mathbb{E}[U_l] = \alpha\phi + 1 - \phi. \tag{34}$$

Case 2: If $1 < R_d \leq \frac{1}{\phi+(1-\phi)P_C^*(b)}$, we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors withdraw early only in the good state. Note that this case is well-defined since $P_C^*(b) < 1$ implies $1 < \frac{1}{\phi+(1-\phi)P_C^*(b)}$.

If the economic state in period 1 is good, then there is no equilibrium in which the bank does not default since the payoff from withdrawing late cannot be greater than the bank's return of 1, which is less than the payoff from withdrawing early.

If the economic state in period 1 is bad, a mass ϕ of liquidity-shocked depositors withdraws in period 1 and a mass $1-\phi$ of normal depositors withdraws in period 2, then the expected payment from withdrawing late is $\frac{(1-R_d\phi)/P_C^*(b)}{1-\phi} \geq R_d$. Therefore withdrawing late is a best response for a late depositor. The best response for a liquidity-shocked depositor is to withdraw early since

$$\begin{aligned}\frac{(1-R_d\phi)/P_C^*(b)}{1-\phi} &< \alpha\frac{1-\phi R_d}{1-\phi}, \text{ since } P_C^*(b) > \frac{1}{\alpha} \\ &< \alpha, \text{ since } R_d > 1\end{aligned} \tag{35}$$

Note that this is less than the payoff early depositors receive when withdrawing early, which is αR_d if $R_d \leq \kappa$ or $\alpha\kappa + R_d - \kappa$ if $R_d > \kappa$.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\mathbb{E}[U_l] = \eta\left(\alpha\phi + 1 - \phi\right) + (1-\eta)\left(\alpha R_d\phi + \frac{1-R_d\phi}{P_C^*(b)}\right).$$

The locally optimal R_d is the upper bound of $\frac{1}{\phi+(1-\phi)P_C^*(b)}$ since $\alpha P_C^*(b) > 1$. The maximum expected utility is then

$$\mathbb{E}\left[U_l | R_d = \frac{1}{\phi + (1-\phi)P_C^*(b)}\right] = \eta\left(\alpha\phi + 1 - \phi\right) + (1-\eta)\frac{\alpha\phi + 1 - \phi}{\phi + (1-\phi)P_C^*(b)}. \tag{36}$$

Since $P_C^*(b) < 1$, it is clear to see that the expected utility from Case 2 (equation (36)) is greater than the expected utility from Case 1 (equation (34)).

Case 3: If $\frac{1}{\alpha P_C^*(b)(1-\phi)+\phi} \leq R_d \leq 1$, we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.[33] Note that this case is well-defined since $\alpha P_C^*(b) > 1$ implies $\frac{1}{\alpha P_C^*(b)(1-\phi)+\phi} < 1$.

If the economic state in period 1 is ω, a mass ϕ of liquidity-shocked depositors withdraws in period 1 and a mass $1-\phi$ of normal depositors withdraws in period 2, then the expected payment from withdrawing late is $\frac{(1-R_d\phi)/P_C^*(\omega)}{1-\phi} > 1 \geq R_d$, which implies that the best response for an individual normal depositor is to withdraw late. The best response for a liquidity-shocked depositor is to withdraw early since $\frac{(1-R_d\phi)/P_C^*(\omega)}{1-\phi} \leq \alpha R_d$. Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\mathbb{E}[U_l] = \eta\left(\alpha R_d\phi + 1 - R_d\phi\right) + (1-\eta)\left(\alpha R_d\phi + \frac{1-R_d\phi}{P_C^*(b)}\right).$$

The locally optimal R_d is the upper bound of 1 since $\alpha > 1$ and $\alpha P_C^*(b) > 1$. The maximum expected utility is then

$$\mathbb{E}[U_l | R_d = 1] = \eta\left(\alpha\phi + 1 - \phi\right) + (1-\eta)\left(\alpha\phi + \frac{1-\phi}{P_C^*(b)}\right). \tag{37}$$

Since $\alpha P_C^*(b) > 1$ and $P_C^*(b) < 1$, the expected utility from Case 2 (equation (36)) is greater than the expected utility from Case 3 (equation (37)).

Case 4: If $R_d \leq \frac{1}{\alpha P_C^*(b)(1-\phi)+\phi}$, then there is no equilibrium in which liquidity-shocked depositors withdraw early in the bad state.[34] If liquidity-shocked depositors

[33] Note that under this condition there is no equilibrium in which normal depositors withdraw early. In particular, the bank cannot default in period 1 even if all depositors withdraw early. As a result, if all other depositors withdraw early, then the best response for an individual depositor is to withdraw late because the individual payoff is infinite.

[34] There is no equilibrium in which liquidity-shocked depositors withdraw early in the bad state since the utility from withdrawing early αR_d is less than the payment from withdrawing late (conditional on the other liquidity-shocked depositors withdrawing early), $\frac{(1-R_d\phi)/P_C^*(b)}{1-\phi}$. Moreover, to have an equilibrium in which liquidity-shocked depositors withdraw late, the payment from withdrawing late (conditional on the other liquidity-shocked depositors withdrawing late), $1/P_C^*(b)$, must be larger than the payment from withdrawing early, αR_d, which requires $R_d \leq \frac{1}{\alpha P_C^*(b)}$. Note that $\frac{1}{\alpha P_C^*(b)}$ is less than the bound $\frac{1}{\alpha(1-\phi)P_C^*(b)+\phi}$

withdraw late in the bad state, then the utility of the bank in the bad state, $\frac{1}{P_C^*(b)}$, is less than the utility in the bad state from Case 2 from equation (36) since $\alpha P_C^*(b) > 1$. Similarly, if liquidity-shocked depositors withdraw late in the good state then the utility of the bank in the good state, 1, is less than the utility in the good state from Case 2 from equation (36) since $\alpha > 1$. Therefore Case 3 is dominated by Case 2.

Since Case 2 dominates Case 1, Case 3, and Case 4, the globally optimal early repayment R_d is the local optimum from Case 2, $R_d = \frac{1}{\phi+(1-\phi)P_C^*(b)}$, and the maximum utility is given by equation (36).

Now, the portfolio from Case 2 is dominated by investing a fraction $1 - L$ of the bank's assets in complex assets and setting $R_d = L$. To see this, consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.[35] If the economic state in period 1 is ω, a mass ϕ of liquidity-shocked depositors withdraws in period 1, and a mass $1 - \phi$ of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is $\frac{(L-\phi L)/P_C^*(\omega)+\mu_\omega R(1-L)}{1-\phi} > L = R_d$. Moreover, $\frac{(L-\phi L)/P_C^*(\omega)+\mu_\omega R(1-L)}{1-\phi} < \alpha L = \alpha R_d$ by $\frac{1}{P_C^*(b)} < R$ and the assumption in (14) (when the minimum is the right term), which implies that the best response for a liquidity-shocked depositor is to withdraw early. Therefore, there is an equilibrium as described. In this equilibrium the utility of the bank is

$$\mathbb{E}[U_C|R_d = L] = \eta\left(\alpha L\phi + L - \phi L + \mu_g R(1-L)\right) + (1-\eta)\left(\alpha L\phi + \frac{L-\phi L}{P_C^*(b)} + \mu_b R(1-L)\right). \tag{38}$$

since $\alpha P_C^*(b) > 1$. Hence, there is no equilibrium for $R_d \in \left[\frac{1}{\alpha P_C^*(b)}, \frac{1}{\alpha(1-\phi)P_C^*(b)+\phi}\right]$.

[35] Note that there does not exist an equilibrium in which normal depositors withdraw early by similar reasoning as in Case 3 above.

Then, subtracting (36) from (38) obtains

$$
\begin{aligned}
&(1-L)\eta(\mu_g R - \{\alpha\phi + 1 - \phi\}) + (1-\eta)\mu_b R(1-L) \\
&+ (1-\eta)\frac{L[\alpha\phi^2 P_C^*(b) + \phi(1-\phi) + \alpha\phi(1-\phi)P_C^*(b) + (1-\phi)^2 P_C^*(b)] - P_C^*(b)(\alpha\phi + 1 - \phi)}{P_C^*(b)(\phi + (1-\phi)P_C^*(b))} \\
&> (1-L)\eta(\mu_g R - \{\alpha\phi + 1 - \phi\}) + (1-\eta)\mu_b R(1-L) \\
&+ (1-\eta)\frac{L[\alpha\phi^2 P_C^*(b) + \phi(1-\phi)P_C^*(b) + \alpha\phi(1-\phi)P_C^*(b) + (1-\phi)^2 P_C^*(b)] - P_C^*(b)(\alpha\phi + 1 - \phi)}{P_C^*(b)(\phi + (1-\phi)P_C^*(b))}, \\
&\text{ since } P_C^*(b) < 1 \\
&= (1-L)\eta(\mu_g R - \{\alpha\phi + 1 - \phi\}) + (1-\eta)(1-L)\left(\mu_b R - \frac{\alpha\phi + 1 - \phi}{\phi + (1-\phi)P_C^*(b)}\right) \\
&> (1-L)\eta(\mu_g R - \{\alpha\phi + 1 - \phi\}) + (1-\eta)(1-L)(\mu_b R - \alpha), \text{ since } \alpha P_C^*(b) > 1 \\
&> 0 \text{ by } (33)
\end{aligned} \tag{39}
$$

This shows that holding only liquid assets is a dominated portfolio.

Part 2: In Part 2, we show that banks have no incentive to hold excess liquidity conditional on investing in either complex or simple assets and maintaining a debt contract of $R_d = \kappa$.

Complex banks. As shown in the proof of Lemma 1, the utility of a complex bank given the debt contract of $R_d = \kappa$, the bank-run conditions as described in Proposition 2, and an asset price of $P_C(b)$ is given by equation (2). It suffices to show that at the equilibrium price $P_C^*(b)$ we have $\frac{\partial \mathbf{E}[U_C|P_C^*(b)]}{\partial L} < 0$. Note that

$$
\frac{\partial \mathbf{E}[U_C|P_C^*(b)]}{\partial L} = \eta(1-\mu_g R) + (1-\eta)(\alpha\phi + 1 - \phi)(1 - P_C(b)\mu_b R). \tag{40}
$$

Since the equilibrium price satisfies $\frac{1}{\alpha} < P_C^*(b)$, we have that

$$
\frac{\partial \mathbf{E}[U_C|P_C^*(b)]}{\partial L} < \eta(\alpha\phi + 1 - \phi - \mu_g R) + (1-\eta)(\alpha\phi + 1 - \phi)\left(1 - \frac{1}{\alpha}\mu_b R\right) \tag{41}
$$

$$
= \eta(\alpha\phi + 1 - \phi - \mu_g R) + (1-\eta)\frac{\alpha\phi + 1 - \phi}{\alpha}(\alpha - \mu_b R). \tag{42}
$$

Observe that $\alpha\phi + 1 - \phi - \mu_g R < \alpha - \mu_b R$. Therefore, if $\alpha - \mu_b R < 0$, then we obtain $\frac{\partial \mathbf{E}[U_C|P_C^*(b)]}{\partial L} < 0$. If $\alpha - \mu_b R > 0$, then since $\frac{\alpha\phi+1-\phi}{\alpha} < 1$ we get that

$$\frac{\partial \mathbf{E}[U_C|P_C^*(b)]}{\partial L} < \eta(\alpha\phi + 1 - \phi - \mu_g R) + (1-\eta)(\alpha - \mu_b R) < 0 \text{ by (33)} \tag{43}$$

Simple banks. As shown in the proof of Lemma 1, the utility of a complex bank given the debt contract of $R_d = \kappa$, the bank-run conditions as described in Proposition 2, and an asset price of $P_C(b)$ is given by equation (3). It suffices to show that at the equilibrium price $P_C^*(b)$ we have $\frac{\partial \mathbf{E}[U_S|P_C^*(b)]}{\partial L} < 0$. Note that

$$\begin{aligned}\frac{\partial \mathbf{E}[U_S|P_C^*(b)]}{\partial L} &= \eta(\mu_g(1-R) + (1-\mu_g)(\alpha\phi + 1 - \phi)) \\ &\quad + (1-\eta)\left(\mu_b\left(\frac{1}{P_C(b)} - R\right) + (1-\mu_b)(\alpha\phi + 1 - \phi)\right).\end{aligned} \tag{44}$$

Since the equilibrium price satisfies $\frac{1}{\alpha} < P_C^*(b)$, we have that

$$\begin{aligned}\frac{\partial \mathbf{E}[U_S|P_C^*(b)]}{\partial L} &< \eta(\mu_g(1-R) + (1-\mu_g)(\alpha\phi + 1 - \phi)) \\ &\quad + (1-\eta)\left(\mu_b(\alpha - R) + (1-\mu_b)(\alpha\phi + 1 - \phi)\right).\end{aligned} \tag{45}$$

Since $\alpha > \alpha\phi + 1 - \phi > 1$, we have

$$\begin{aligned}\frac{\partial \mathbf{E}[U_S|P_C^*(b)]}{\partial L} &< \eta(\mu_g(\alpha\phi + 1 - \phi - R) + (1-\mu_g)(\alpha\phi + 1 - \phi)) \\ &\quad + (1-\eta)\left(\mu_b(\alpha - R) + (1-\mu_b)\alpha\right) \\ &= \eta(\alpha\phi + 1 - \phi - \mu_g R) + (1-\eta)(\alpha - \mu_b R) \\ &< 0 \text{ by (33)}\end{aligned} \tag{46}$$

□

Lemma 5 (Complex-asset price (endogenous V^*))**.** *If*

$$\frac{\eta(1-\mu_g)\phi(\kappa - L)(\alpha - 1) + (1-\eta)\mu_b(1 - \phi - \alpha\phi(\kappa - 1))}{(1-\eta)\mu_b(1-\phi\kappa)} < R, \tag{47}$$

then, at the equilibrium volume of complex banks V^*, the complex-asset price in bad times satisfies $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < 1$.

Proof. Before going into the proof, we first explain the intuition behind the parametric restriction. In particular, (47) implies that the return to long-term assets, R, is high enough that it exceeds the return from buying complex assets in the interbank market, $\frac{1}{P_C^*(b)}$. In particular, the result in the lemma implies that the return from buying complex assets in the interbank market, $\frac{1}{P_C^*(b)}$, is between the return on liquid assets, 1, and the return on long-term assets, R.

As for the proof itself, first recall the relative advantage of investing in complex assets compared to simple assets Δ as defined in equation (4). Since banks are ex-ante identical, the price must be such that banks are indifferent between complex and simple assets in an equilibrium. Note that there is no equilibrium in which banks invest in only complex or simple assets. If all banks invested in simple assets, then the price would be equal to 1, but in that case banks would prefer to invest in complex assets. If all banks invested in complex assets, then the price would be equal to zero, but in that case banks would prefer to invest in simple assets.

At the maximum possible price $P_C(b) = 1$, then Δ in equation (4) is equal to $\eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) + (1 - \eta)\mu_b\phi(\alpha - 1)[L + R(1 - L) - \kappa]$, which is positive by the second inequality in (15). Therefore, complex banks have a higher expected utility.

If the price is very low, then it is easy to see that simple banks have a higher expected utility. Specifically, if the price is as low as $P_C(b) = \frac{1}{R}$ then the difference in utility Δ expressed in equation (4) at $P_C(b) = \frac{1}{R}$ is equal to

$$\Delta\left(\frac{1}{R}\right) = \eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) + (1 - \eta)\mu_b[1 - \phi + \alpha\phi - R(1 - \phi\kappa) - \alpha\phi\kappa],$$

which is negative by (47).

Since the relative benefit of complex assets $\Delta(P_C(b))$ is increasing and continuous with $\Delta\left(\frac{1}{R}\right) < 0 < \Delta(1)$, there is a unique equilibrium price that satisfies $\frac{1}{R} < P_C^*(b) < 1$.

Finally, the same argument from the proof of Corollary 2 implies $\frac{1}{\alpha} < \frac{1}{R}$. □

Proof of Proposition 3

Proposition 3. *If η is sufficiently large, the equilibrium complex-asset price in bad times $P_C^*(b)$ is increasing in the liquidity level L.*

Proof. Note that the term of $\frac{\partial\Delta}{\partial L}$ (from equation (5)) corresponding to the bad state, $(1 - \eta)\mu_b(P_C^*(b)R - 1)\left[-(\alpha\phi + 1 - \phi) + \frac{1}{P_C^*(b)}\right]$, is greatest when $P_C^*(b) = (R(\alpha\phi + 1 - \phi))^{-\frac{1}{2}}$. Therefore the maximum possible value for $\frac{\partial\Delta}{\partial L}$ occurs at this value of $P_C^*(b)$, and it is equal to

$$-\eta(1-\mu_g)\phi(\alpha-1)+(1-\eta)\mu_b\left(R^{\frac{1}{2}}-(\alpha\phi+1-\phi)^{\frac{1}{2}}\right)^2 \tag{48}$$

Then note that this is negative for η sufficiently large:

$$\eta > \frac{\left(R^{\frac{1}{2}}-(\alpha\phi+1-\phi)^{\frac{1}{2}}\right)^2\mu_b}{\left(R^{\frac{1}{2}}-(\alpha\phi+1-\phi)^{\frac{1}{2}}\right)^2\mu_b+\phi(\alpha-1)(1-\mu_g)}. \tag{49}$$

□

Proof of Proposition 4

Proposition 4 (Welfare-maximizing volume of complex banks)**.** *There exists $\hat{L}$ such that the following hold. When liquidity requirements are tight, $L > \hat{L}$, then there is excess investment in complex assets, i.e., $V^W < V^*$. Moreover, the welfare-maximizing complex-asset price in bad times is equal to the maximum level of 1, i.e., $P_C^W(b) = 1 > P_C^*(b)$. When liquidity requirements are loose, $L < \hat{L}$, then there is underinvestment in complex assets, i.e., $V^W > V^*$. Moreover, the welfare-maximizing complex-asset price in bad times $P_C^W(b)$ satisfies $0 < P_C^W(b) < P_C^*(b)$.*

Proof. Consider the total effect of varying the volume of complex banks on welfare

$$\frac{d\mathcal{W}}{dV^W} = \frac{\partial\mathcal{W}}{\partial V^W} + \frac{\partial\mathcal{W}}{\partial P_C^W(b)}\frac{\partial P_C^W(b)}{\partial V^W}.$$

The first term corresponds to the direct effect and is equal to the relative advantage of investing in complex assets Δ as defined in equation (4). The second term corresponds to the indirect effect through the adjustment of price, which is not internalized by the banks in the equilibrium. Since $P_C^W(b)$ is related to V^W by the market-clearing condition in equation (1), we have that the volume of complex banks is inversely related to the price:

$$\frac{\partial P_C^W(b)}{\partial V^W} = -\frac{L-\kappa\phi}{(1-L)R(V^W)^2} = -\frac{P_C^W(b)}{V^W(1-V^W)} < 0. \tag{50}$$

The price, in turn, affects welfare as follows:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)} = (1-\eta)\mu_b \left[V^W(\alpha\phi + 1 - \phi)R(1-L) - (1-V^W)\frac{L-\kappa\phi}{P_C^W(b)^2} \right]$$

The first term represents the marginal benefit of increasing the price in terms of supporting complex banks, which sell assets, while the second term represents the marginal cost in terms of decreasing the return for simple banks that draw a high return, which buy assets.

Then, we can write:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)} \frac{\partial P_C^W(b)}{\partial V^W} = -(1-\eta)\mu_b \left[\frac{1}{1-V^W}(\alpha\phi + 1 - \phi)P_C^W(b)R(1-L) - \frac{1}{V^W}\frac{L-\kappa\phi}{P_C^W(b)} \right].$$

Therefore, the total effect on welfare can be written as

$$\begin{aligned}
\frac{d\mathcal{W}}{dV^W} = {} & \eta(1-\mu_g)\phi(\kappa - L)(\alpha - 1) \\
& + (1-\eta)\mu_b \left[(\alpha\phi + 1 - \phi)\left(L + P_C^W(b)R(1-L)\left(1 - \frac{1}{1-V^W}\right)\right)\right] \\
& - (1-\eta)\mu_b \left[\alpha\kappa\phi + \frac{L-\kappa\phi}{P_C^W(b)}\left(1 - \frac{1}{V^W}\right) + R(1-L) \right].
\end{aligned}$$

This first term shows that the planner's incentive to invest in complex assets is increasing in the liquidity services advantage of complex assets in good times. The second term corresponds to the advantage of complex banks in bad times compared to simple banks that default, which is their ability to sell assets in the interbank market. This advantage directly increases the planner's incentive to invest in complex assets, but the planner also internalizes the fact that increasing the volume of complex banks leads to a price reduction that offsets this advantage. The third term corresponds to the disadvantage of complex banks in bad times compared to simple banks with a high return, which is the fact that they always experience a run. This disadvantage directly decreases the planner's incentive to invest in complex assets, but the planner also internalizes the fact that increasing the volume of complex banks leads to a price reduction that has the benefit of increasing the return of the simple banks.

Further simplifying by substituting the market-clearing condition in equation (1) obtains the following:

$$\frac{d\mathcal{W}}{dV^W} = \eta(1-\mu_g)\phi(\kappa - L)(\alpha-1) - (1-\eta)\mu_b\phi\kappa(1-\phi)(\alpha-1)$$
$$= \quad (\alpha-1)\phi\eta(1-\mu_g)\left[\underbrace{\kappa\left(1-\frac{(1-\phi)(1-\eta)\mu_b}{\eta(1-\mu_g)}\right)}_{=\hat{L}} - L\right]. \tag{51}$$

This equation illustrates that the direct and price effects of increasing the volume of complex banks in bad times offset each other such that the net effect is constant in the volume of complex banks and the price. This drives the solution to the boundaries of the choice space depending on the sign of equation (51), which in turn depends on the magnitude of the tightness of liquidity requirements L relative to $\hat{L}$.

Case 1: If $L > \hat{L}$, the optimal policy is to reduce the volume of complex banks and therefore increase the complex-asset price $P_C^W(b)$ until it is equal to its upper bound of 1. Note that reducing the volume of complex banks does not create an incentive to deviate from the debt contract $R_d = \kappa$ and bank-run conditions as described in Proposition 2. In particular, the proof of Lemma 1 shows that the debt contract and bank-run conditions hold for any $P_C(b)$ satisfying $\frac{1}{R} \leq P_C(b) \leq 1$. Since $\frac{1}{R} < P_C^*(b) < 1$, they hold for $P_C^W(b) \in \left[P_C^*(b), 1\right]$.

Intuitively, increasing the asset price supports complex banks since they experience a run in the bad state. Once the complex-asset price is equal to 1, which is the return on liquid assets, there is no incentive to further reduce the volume of complex banks since the price cannot be any higher. Additionally, reducing the volume of complex banks has a cost since complex assets yield a higher expected utility when $P_C^W(b) = 1$.[36]

Case 2: If $L < \hat{L}$, the optimal policy is to increase the volume of complex banks and therefore decrease the complex-asset price $P_C^*(b)$ until the debt contract of $R_d = \kappa$ and bank-run conditions as described in Proposition 2 can no longer be maintained.

The minimal $P_C^W(b)$ such that banks still prefer to offer an early repayment of $R_d = \kappa$ is strictly greater than 0. To see this, note that Lemma 1 shows that a necessary condition for banks to set $R_d = \kappa$ is given by (23). Note that the left-hand side of (23) has the same

[36]This cost can be seen by differentiating the expected utility for $P_C^W(b) = 1$:

$$\text{sign}\left(\frac{\partial \mathcal{W}}{\partial V^W}\right) = \text{sign}\Bigg((1-\eta)\mu_b(L - \kappa + R(1-L)) + \eta(1-\mu_g)(\kappa - L)\Bigg) > 0.$$

sign as when it is multiplied by $P_C(b) > 0$, which can be denoted by

$$F(P_C(b)) \equiv P_C(b)\eta\phi(\alpha - 1)(\kappa - \{L + \mu_b P_C(b) R(1-L)\}) \\ - (1-\eta)(1-\phi)(L + P_C(b)\mu_b R(1-L))(1 - P_C(b)).$$

It is then straightforward to see that $F(0) < 0$, and $0 < F(1)$ follows from the assumption in the first inequality in (15). Since F is quadratic, this implies there is a unique solution $P_C^W(b) \in (0,1)$ to the equation $F(P_C^W(b)) = 0$.

Alternative intuition. Consider the *local* incentive for the planner to increase the volume of complex banks relative to the equilibrium:

$$\frac{d\mathcal{W}}{dV^W}|_{V^W=V^*} = \frac{\partial \mathcal{W}}{\partial V^W}|_{V^W=V^*} + \frac{\partial \mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)} \frac{\partial P_C^W(b)}{\partial V^W}|_{V^W=V^*}.$$

Note that $\frac{\partial \mathcal{W}}{\partial V^W}|_{V^W=V^*} = 0$ since banks are indifferent between the two types of assets in the equilibrium. Recall that $\frac{\partial P_C^W(b)}{\partial V^W} < 0$ from equation (50). Therefore, $\frac{\partial \mathcal{W}}{dV^W}|_{V^W=V^*}$ has the opposite sign as $\frac{\partial \mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)}$, which corresponds to the regulator's incentive to adjust the price that is not internalized by the individual banks:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)} = (1-\eta)\mu_b \left[(\alpha\phi + 1 - \phi) R(1-L) V^* - \frac{(1-V^*)(L-\kappa\phi)}{P_C^*(b)^2} \right].$$

As described above, the first term corresponds to the marginal benefit of increasing the price in terms of increasing the return of complex banks, which sell assets, while the second term corresponds to the marginal cost in terms of decreasing the return for simple banks that draw a high return, which buy assets.

Using the market-clearing condition in equation (1), we can write:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)} = \frac{(1-\eta)\mu_b(L-\kappa\phi)R(1-L)}{L - \kappa\phi + P_C^*(b)(1-L)R} \left[(\alpha\phi + 1 - \phi) - \frac{1}{P_C^*(b)} \right].$$

Therefore, $\frac{\partial \mathcal{W}}{\partial V^W}|_{V^W=V^*}$ has the same sign as

$$(\alpha\phi + 1 - \phi) - \frac{1}{P_C^*(b)}. \tag{52}$$

Note that the equilibrium price at $\hat{L} = \kappa\left(1 - \frac{(1-\phi)(1-\eta)\mu_b}{\eta(1-\mu_g)}\right)$ is equal to $\frac{1}{\alpha\phi+1-\phi}$. If the equilibrium price is increasing in L (see Proposition 3 for a sufficient condition), the expression in (52) is positive (negative) when L is greater (less) than $\hat{L}$. Intuitively, L determines whether the equilibrium price is large enough that the benefit of further increasing the price in terms of insuring complex banks exceeds the cost in terms of decreasing the returns on asset purchases for simple banks.

□

Proof of Proposition 5

Proposition 5 (Welfare-maximizing tightness of liquidity requirements)**.** *If a policymaker can implement the efficient level of investment in complex assets, then the optimal tightness of liquidity requirements is no greater than $\hat{L}$. If a policymaker allows the volume of complex assets to be determined in equilibrium, then the optimal tightness of liquidity requirements is also no greater than $\hat{L}$.*

Proof. **Case 1**: Suppose the policy-maker implements the optimal degree of investment in complex assets, V^W. By the envelope theorem, we have

$$\frac{d\mathcal{W}}{dL} = V^W \frac{\partial \mathbb{E}\left[U_C|P_C^W(b)\right]}{\partial L} + \left(1 - V^W\right) \frac{\partial \mathbb{E}\left[U_S|P_C^W(b)\right]}{\partial L}. \tag{53}$$

Suppose $L > \hat{L}$. Then by Proposition 4, $P_C^W(b) = 1$. In particular, this implies $\frac{1}{\alpha} < P_C^W(b)$. Hence, the proof of Part 2 of Lemma 4 can be applied to show that $\frac{\partial \mathbb{E}\left[U_C|P_C^W(b)\right]}{\partial L} < 0$ and $\frac{\partial \mathbb{E}\left[U_S|P_C^W(b)\right]}{\partial L} < 0$, which implies $\frac{d\mathcal{W}}{dL} < 0$. This implies $L \leq \hat{L}$.

Case 2: Suppose the policy-maker faces the equilibrium degree of investment in complex assets, V^*. Note that

$$\frac{d\mathcal{W}}{dL} = \frac{\partial \mathcal{W}}{\partial L} + \frac{\partial \mathcal{W}}{\partial V^*}\frac{\partial V^*}{\partial L} + \frac{\partial \mathcal{W}}{\partial P_C^*(b)}\frac{\partial P_C^*(b)}{\partial L}. \tag{54}$$

The proof of Part 2 of Part 2 of Lemma 4 shows that $\frac{\partial \mathbb{E}\left[U_C|P_C^*(b)\right]}{\partial L} < 0$ and $\frac{\partial \mathbb{E}\left[U_S|P_C^*(b)\right]}{\partial L} < 0$. This implies

$$\frac{\partial \mathcal{W}}{\partial L} = V^* \frac{\partial \mathbb{E}\left[U_C|P_C^*(b)\right]}{\partial L} + (1 - V^*) \frac{\partial \mathbb{E}\left[U_S|P_C^*(b)\right]}{\partial L} < 0. \tag{55}$$

Note that

$$\frac{\partial \mathcal{W}}{\partial V^*} = \mathbb{E}\left[U_C|P_C^*(b)\right] - \mathbb{E}\left[U_S|P_C^*(b)\right] = 0 \tag{56}$$

since an equilibrium condition is that banks are indifferent between complex and simple assets. Hence $\frac{\partial \mathcal{W}}{\partial V^*}\frac{\partial V^*}{\partial L} = 0$.

By Proposition 3, $\frac{\partial P_C^*(b)}{\partial L} > 0$, so the term $\frac{\partial \mathcal{W}}{\partial P_C^*(b)}\frac{\partial P_C^*(b)}{\partial L}$ has the same sign as $\frac{\partial \mathcal{W}}{\partial P_C^*(b)}$. If $L < \hat{L}$, the proof of Proposition 4 shows that $\frac{\partial \mathcal{W}}{\partial P_C^*(b)} < 0$. This, combined with the signs of the other terms, implies that $\frac{d\mathcal{W}}{dL} < 0$. Hence, the optimal L is either the minimum possible value or greater than $\hat{L}$. To show it is the former, it is convenient to denote welfare as a function of both L and V, or $\mathcal{W}(L,V)$. Let $V^*(L)$ denote the equilibrium volume of complex assets as a function of L, and let $V^W(L)$ denote the optimal volume of complex assets as a function of L. Then we have

$$\begin{aligned}
\mathcal{W}(L=\min, V^*(\min)) &> \mathcal{W}(L=\hat{L}, V^*(\hat{L})) \text{ by the preceding paragraph} && (57)\\
&= \mathcal{W}(L=\hat{L}, V^W(\hat{L})) \text{ by the proof of Proposition 4} && (58)\\
&> \mathcal{W}(L>\hat{L}, V^W(L)) \text{ by Case 1} && (59)\\
&> \mathcal{W}(L>\hat{L}, V^*(L)) \text{ by Proposition 4.} && (60)
\end{aligned}$$

□

Proof of Proposition 6

Rearranging equation (8) and supposing that the volume of complex banks is equal to the level corresponding to no tax $V(0)$, the price can be expressed as

$$P_C^\tau(b) = \frac{\mu_b(L-\kappa\phi)(1-V(0)) + \tau\delta}{RV(0)(1-L)\mu_b},$$

hence it follows that

$$\frac{\partial P_C^\tau(b)}{\partial \tau} = \frac{\delta}{RV(0)(1-L)\mu_b} > 0. \tag{61}$$

Note also that increasing the tax maintains the debt contract and bank-run conditions as stated in Proposition 2. In particular, the proof of Lemma 1 shows that the debt contract and bank-run conditions hold for any $P_C(b)$ satisfying $\frac{1}{R} \le P_C(b) \le 1$. Since the price that would occur in the absence of a tax $P_C^0(b)$ satisfies $\frac{1}{R} \le P_C^0(b) < 1$, this implies that they hold for $P_C^\tau(b) \in \left[P_C^0(b), 1\right]$.[37]

[37]Note that we restrict to δ small enough such that a similar proof as in Lemma 5 works to show that there exists a unique equilibrium price $P_C^\tau(b) \in \left(\frac{1}{R}, 1\right)$.

Proof of Proposition 7

Proposition 7 (Unanticipated QE). *If QE is unanticipated, then the optimal tax is positive and equal to the minimum of income ν and the minimum tax necessary to increase the complex-asset price in bad times $P_C^\tau(b)$ to 1.*

Proof. Rearranging the market-clearing condition (equation (8)), we substitute

$$\frac{\tau\delta}{P_C^\tau(B)} = V(0)R(1-L)\mu_b - \Big(1 - V(0)\Big)\ \mu_b\ \frac{L-\kappa\phi}{P_C^\tau(b)}$$

into the expression for welfare (equation (11)). Then, if $P_C^\tau(b) < 1$, taking the derivative with respect to τ and using the expression for $\frac{\partial P_C^\tau(b)}{\partial \tau}$ from equation (61) in the proof of Proposition 6 obtains

$$\begin{aligned}\frac{\partial \mathcal{W}(\tau)}{\partial \tau} &= V(0)(1-\eta)(\alpha\phi + 1 - \phi)\Big[\underbrace{\frac{\delta}{RV(0)(1-L)\mu_b}}_{=\frac{\partial P_C^\tau(b)}{\partial \tau}}\mu_b R(1-L) - \delta\Big] \\ &\quad - (1-V(0))(1-\eta)[\mu_b\delta + (1-\mu_b)(\alpha\phi + 1 - \phi)\delta] \\ &> (1-\eta)\delta(\alpha\phi + 1 - \phi)\left[V(0)\left(\frac{1}{V(0)} - 1\right) - (1 - V(0))\right] \\ &= 0.\end{aligned}$$

Since welfare is increasing in τ as long as $P_C^\tau(b) < 1$, the government optimally increases taxes until either $P_C^\tau(b) = 1$ or $\tau = \nu$. □

Proof of Proposition 8

Proposition 8. *If QE is anticipated and $P_C^\tau(b) < 1$, then*

(a) the equilibrium complex-asset price is increasing in the tax τ: $\frac{\partial P_C^\tau(b)}{\partial \tau} > 0$, and

(b) the equilibrium volume of complex banks is increasing in the tax τ: $\frac{\partial V(\tau)}{\partial \tau} > 0$.

Proof. **Part (a).** Note that the relative benefit of investing in complex assets is summarized by subtracting equation (10) from equation (9):

$$\begin{aligned}
H^{\tau}(P_C(b)) &\equiv \mathbb{E}[U_C|P_C(b)] - \mathbb{E}[U_S|P_C(b)] \\
&= \eta(1-\mu_g)\phi(\kappa - L)(\alpha - 1) \\
&\quad + (1-\eta)\mu_b \left[(\alpha\phi + 1 - \phi)(L + P_C(b)R(1-L)) - \left(\alpha\kappa\phi + \frac{L-\kappa\phi}{P_C(b)} + R(1-L)\right)\right] \\
&\quad + \delta\phi(\alpha - 1)[(1-\eta)(\nu - \tau)\mu_b - \eta\nu(1-\mu_g)]. \qquad (62)
\end{aligned}$$

Since banks are ex-ante identical and anticipate the tax, the equilibrium price must be such that banks are indifferent between complex and simple assets. For δ sufficiently small, there is a unique price $P_C^{\tau}(b) \in \left(\frac{1}{R}, 1\right)$ satisfying $H^{\tau}(P_C(b)) = 0$.[38] Then, by the implicit function theorem, we have:

$$\begin{aligned}
\frac{\partial P_C^{\tau}(b)}{\partial \tau} &= -\frac{\partial H/\partial \tau}{\partial H/\partial P_C(b)} \\
&= \frac{\delta\phi(\alpha - 1)}{(\alpha\phi + 1 - \phi)R(1-L) + \frac{L-\kappa\phi}{P_C^{\tau}(b)^2}} > 0. \qquad (63)
\end{aligned}$$

Note that increasing the tax maintains the debt contract and bank-run conditions as stated in Proposition 2. In particular, the proof of Lemma 1 shows that the debt contract and bank-run conditions hold for any $P_C(b)$ satisfying $\frac{1}{R} \leq P_C(b) \leq 1$. Since the price that would occur in the absence of a tax $P_C^0(b)$ satisfies $\frac{1}{R} \leq P_C^0(b) < 1$, this implies that they hold for $P_C^{\tau}(b) \in \left[P_C^0(b), 1\right]$.

Part (b). Rearranging equation (8) implies that

$$V(\tau) = \frac{\mu_b\left(L - \kappa\phi\right) + \delta\tau}{P_C^{\tau}(b)\mu_b R(1-L) + \mu_b(L-\kappa\phi)}. \qquad (64)$$

[38] Note that we restrict to δ small enough such that a similar proof as in Lemma 5 works to show that there exists a unique equilibrium price $P_C^{\tau}(b) \in \left(\frac{1}{R}, 1\right)$.

Therefore, using equation (63) we have:

$$\frac{\partial V(\tau)}{\partial \tau} = \frac{\delta(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L-\kappa\phi)) - (\mu_b(L-\kappa\phi)+\delta\tau)\frac{\partial P_C^\tau(b)}{\partial \tau}\mu_b R(1-L)}{\left(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L-\kappa\phi)\right)^2}$$
$$= \frac{\delta(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L-\kappa\phi)) - (\mu_b(L-\kappa\phi)+\delta\tau)\frac{\delta\phi(\alpha-1)\mu_b R(1-L)}{(\alpha\phi+1-\phi)R(1-L)+\frac{L-\kappa\phi}{P_C^\tau(b)^2}}}{\left(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L-\kappa\phi)\right)^2} > 0, \tag{65}$$

where the inequality follows because

$$\frac{\phi(\alpha-1)R(1-L)}{(\alpha\phi+1-\phi)R(1-L)+\frac{L-\kappa\phi}{P_C^\tau(b)^2}} < 1$$

since $L > \kappa\phi$, and

$$P_C^\tau(b)R(1-L) + L - \kappa\phi = (\delta\tau + \mu_b(L-\kappa\phi))\frac{1}{V(\tau)} \geq \delta\tau + \mu_b(L-\kappa\phi)$$

since $V(\tau) \leq 1$. □

Proof of Proposition 9

Proposition 9 (Anticipated QE)**.** *If QE is anticipated, the optimal tax can in general be either positive or zero. If the liquidity level L is sufficiently high, then the optimal tax is zero.*

Proof. **Proof that the optimal tax is zero when L is large.** Since banks are ex-ante identical and banks anticipate the tax, the equilibrium price must be such that banks are indifferent between complex and simple assets. Therefore it suffices to consider how the tax affects the expected utility from investing in complex assets. Taking the derivative of the expected utility from investing in complex assets (equation (9)) with respect to τ obtains

$$\frac{\partial \mathbb{E}[U_C | P_C(b) = P_C^\tau(b)]}{\partial \tau} = (1-\eta)(\alpha\phi+1-\phi)\left[\underbrace{\frac{\delta\phi(\alpha-1)}{(\alpha\phi+1-\phi)R(1-L)+\frac{L-\kappa\phi}{P_C^\tau(b)^2}}}_{=\frac{\partial P_C^\tau(b)}{\partial \tau}}\mu_b R(1-L) - \delta\right]$$
$$+ (1-\eta)\left[\frac{\delta}{P_C^\tau(b)} - \frac{\tau\delta}{P_C^\tau(b)^2}\frac{\partial P_C^\tau(b)}{\partial \tau}\right]. \tag{66}$$

For L close to 1, this is approximately equal to

$$\frac{\partial \mathbb{E}[U_C|P_C(b) = P_C^\tau(b)]}{\partial \tau} \approx -(1-\eta)(\alpha\phi + 1 - \phi)\delta + (1-\eta)\left[\frac{\delta}{P_C^\tau(b)} - \frac{\tau\delta}{P_C^\tau(b)^2}\frac{\partial P_C^\tau(b)}{\partial \tau}\right]$$
$$< (1-\eta)\delta\left[\frac{1}{P_C^\tau(b)} - (\alpha\phi + 1 - \phi)\right].$$

For this to be negative, it suffices to show $P_C^\tau(b) > \frac{1}{\alpha\phi+1-\phi}$. Recall that $P_C^\tau(b)$ is the unique positive solution $H^\tau(P_C^\tau(b)) = 0$, where $H^\tau(P_C(b))$ in equation (62) is the relative advantage of investing in complex assets for a given price $P_C(b)$. Since H^τ is increasing in $P_C(b)$, to show that $P_C^\tau(b) > \frac{1}{\alpha\phi+1-\phi}$ it suffices to show $H^\tau\left(\frac{1}{\alpha\phi+1-\phi}\right) < 0$. For L near 1 and δ negligibly small compared to the other terms in H^τ, we have:

$$H^\tau\left(\frac{1}{\alpha+1-\phi}\right) \approx \phi(\alpha-1)\left[\eta(1-\mu_g)(\kappa-1) - (1-\eta)\mu_b\kappa(1-\phi)\right] < 0,$$

where the inequality follows from the fact that if L is near 1, then the second inequality in 15 implies that κ must also be close to 1.[39]

Proof that the optimal tax can be positive. Evaluating equation (66) at $t = 0$ and L close to $\kappa\phi$ obtains

$$\frac{\partial \mathbb{E}[U_C|P_C(b) = P_C^\tau(b)]}{\partial \tau}\Big|_{\tau=0} \approx (1-\eta)\delta\left[\frac{1}{P_C^\tau(b)} - (1 + (1-\mu_b)\phi(\alpha-1))\right].$$

For this to be positive, it suffices to show that $P_C^\tau(b) < \frac{1}{\alpha\phi+1-\phi}$. By similar reasoning as above, it suffices to show $H^\tau\left(\frac{1}{\alpha\phi+1-\phi}\right) > 0$. Note that at $L = \kappa\phi$ and for δ negligibly small compared to the other terms in H^τ we have:

$$H^\tau\left(\frac{1}{\alpha\phi+1-\phi}\right) \approx \kappa\phi(1-\phi)(\alpha-1)\left[\eta(1-\mu_g) - (1-\eta)\mu_b\right],$$

which is positive if $\eta(1-\mu_g) > (1-\eta)\mu_b$.[40]

□

[39]Note that the maximum L that is consistent with the parametric restriction in the second inequality in (15) is generally less than 1 and not necessarily large enough for this result to hold assuming the other parameters are held fixed.

[40]Note that the minimum L that is consistent with the parametric restrictions in Proposition 2 is generally greater than $\kappa\phi$ and not necessarily small enough for this result to hold assuming the other parameters are held fixed.

Proof of Proposition 10

Proposition 10 (Ex-ante insurance)**.** *Implementing the ex-ante insurance policy (i) increases the equilibrium complex-asset price in bad times, (ii) decreases the volume of complex banks, and (iii) increases overall welfare.*

Proof. Denote the volume of complex banks when the redistributive policy is in place by V^τ and the equilibrium price by $P_C^\tau(b)$. Note that the optimal debt contract is still given by $R_d = \kappa$. In particular, complex banks offer $R_d = \kappa$ as long as (23) holds. Since it holds with $P_C^*(b)$ and $P_C^\tau(b) > P_C^*(b)$, it also holds with $P_C^\tau(b)$. It is straightforward to see that the incentive for simple banks to offer $R_d = \kappa$ is strengthened when they no longer experience a risk of a run.

Complex-asset price in bad times. First, we show that the policy leads to an increase in the equilibrium price, or $P_C^\tau(b) > P_C^*(b)$. To see this, first note that the expected utility of a complex bank is

$$\begin{aligned}\mathbb{E}[U_C|P_C^\tau(b)] = \eta\Big(\phi\alpha\kappa + L - \phi\kappa + \mu_g R(1-L)\Big) \\ + (1-\eta)\Big(\phi\alpha + 1 - \phi\Big)\Big(L + P_C^\tau(b)\mu_b R(1-L)\Big).\end{aligned} \tag{67}$$

The redistributive tax changes the period-2 income for all simple banks to $\mu_g R(1-L)$, which is high enough to avoid a run by the second inequality in (15). The expected utility for a simple bank is therefore given by

$$\begin{aligned}\mathbb{E}[U_S|P_C^\tau(b)] = \eta\Big(\phi\alpha\kappa + L - \phi\kappa + \mu_g R(1-L)\Big) \\ + (1-\eta)\left(\mu_b\Big(\phi\alpha\kappa + \frac{L-\phi\kappa}{P_C^\tau(b)} + R(1-L)\Big) + (1-\mu_b)(\phi\alpha + 1 - \phi)L\right).\end{aligned} \tag{68}$$

The complex-asset price is determined by the indifference condition:

$$\begin{aligned}0 &= \mathbb{E}[U_C|P_C^\tau(b)] - \mathbb{E}[U_S|P_C^\tau(b)] &(69)\\ &= \left(\frac{1}{P_C^\tau(b)} - (\phi\alpha + 1 - \phi)\right)\Big(L - \phi\kappa + P_C^\tau(b)R(1-L)\Big) + \phi\Big(\alpha - (\phi\alpha + 1 - \phi)\Big). &(70)\end{aligned}$$

Rearranging the right-hand side of (70) obtains

$$\begin{aligned}\Upsilon(P_C^\tau(b)) := &\underbrace{(\phi\alpha+1-\phi)R(1-L)}_{:=a_2}P_C^\tau(b)^2\\ &+\underbrace{\Big((\phi\alpha+1-\phi)L-R(1-L)-\phi\alpha\kappa\Big)}_{:=a_1}P_C^\tau(b)\\ &\underbrace{-(L-\phi\kappa)}_{:=a_0}.\end{aligned} \tag{71}$$

Clearly, $\Upsilon(0)<0<\Upsilon(1)$, meaning that $\Upsilon(\cdot)$ has a unique root in $(0,1)$.[41] So, $P_C^\tau(b)=\frac{-a_1+\sqrt{a_1^2-4a_0a_2}}{2a_2}$. Recall that $P_C^*(b)=\frac{-\tilde{a}_1+\sqrt{\tilde{a}_1^2-4a_0a_2}}{2a_2}$ where $\tilde{a}_1>a_1$ (see Proposition 2). Since $\frac{\partial P_C^*(b)}{\partial \tilde{a}_1}<0$ thus $P_C^\tau(b)>P_C^*(b)$, finishing the proof of the lemma.

Volume of complex banks. Since the equilibrium price increases, the market-clearing condition in equation (1) implies that the volume of complex banks decreases, or $V^\tau<V^*$.

Welfare. Finally, we show the policy improves welfare. Recall that welfare is defined as the expected utility of representative depositor, which is given by

$$\begin{aligned}\mathcal{W}^\tau &= V^\tau\mathbb{E}[U_C|P_C^\tau(b)]+(1-V^\tau)\mathbb{E}[U_S|P_C^\tau(b)]\\ &=(1-V^\tau)\left[(1-\eta)\left(\mu_b\left(\phi\alpha\kappa+\frac{L-\phi\kappa}{P_C^\tau(b)}+R(1-L)\right)+(1-\mu_b)\Big(\phi\alpha+1-\phi\Big)L\right)\right]\\ &\quad+V^\tau\left[(1-\eta)\Big(\phi\alpha+1-\phi\Big)\Big(L+(1-L)P_C^\tau(b)\mu_bR\Big)\right]\\ &\quad+\eta\Big(\phi\alpha\kappa+L-\phi\kappa+\mu_gR(1-L)\Big).\end{aligned} \tag{72}$$

Substituting the market-clearing condition in (1) into (72) obtains

$$\begin{aligned}\mathcal{W}^\tau &= R(1-L)\Big(\eta\mu_g+(1-\eta)\mu_b\Big)+\Big(\eta+(1-\eta)(\phi\alpha+1-\phi)\Big)L+\eta\mu_g\phi(\alpha-1)\kappa\\ &\quad+\phi(\alpha-1)\kappa\left[\eta(1-\mu_g)+(1-V^\tau)\mu_b(1-\eta)(1-\phi)\right].\end{aligned} \tag{73}$$

[41] Note that $\Upsilon(1)=\phi\Big(L+R(1-L)-\kappa\Big)(\alpha-1)>0$ and $\Upsilon(0)=-(L-\phi\kappa)<0$.

Now, recall that welfare in the original equilibrium is

$$\mathcal{W} = R(1-L)\Big(\eta\mu_g + (1-\eta)\mu_b\Big) + \Big(\eta + (1-\eta)\big(\phi\alpha + 1 - \phi\big)\Big)L + \eta\mu_g\phi(\alpha-1)\kappa$$
$$+\,\phi(\alpha-1)\left[V^*\eta\kappa(1-\mu_g) + (1-V^*)\Big(\eta(1-\mu_g)L + (1-\eta)\mu_g(1-\phi)\kappa\Big)\right]. \tag{74}$$

Then finally, equation (73) minus equation (74) is

$$\left((1-V^*)\eta(\kappa-L)(1-\mu_g) + \underbrace{(V^*-V^\tau)}_{>0}(1-\phi)\mu_b(1-\eta)\kappa\right)(\alpha-1)\phi > 0.$$

Thus, the redistributive policy always improves welfare compared to the original equilibrium. □

Proof of Proposition 11

Proposition 11. *If $L < \hat{L}$ and ν is sufficiently large, then the constrained-efficient volume of complex banks can be implemented via anticipated QE. However, the tax that implements the constrained-efficient volume of complex banks may not be welfare-optimizing. If $L > \hat{L}$, then neither QE nor the ex-ante insurance policy can implement the constrained-efficient volume of complex banks.*

Proof. **Case 1:** $L < \hat{L}$**.** In this case, the constrained-efficient volume of complex banks is greater than the equilibrium (Proposition 4). Note that the expression for the volume of complex banks $V(\tau)$ in equation (64) is increasing and unbounded in the tax τ. Therefore, there is a tax level for which $V(\tau)$ is equal to the volume of complex banks in the planner solution V^W, which can be implemented as long as ν is sufficiently high.

Case 2: $L > \hat{L}$**.** In this case, the constrained-efficient volume of complex banks is less than the equilibrium (Proposition 4). Unanticipated QE cannot implement the constrained-efficient volume of complex banks because it has no effect on the volume of complex banks. Anticipated QE cannot implement the constrained-efficient volume of complex banks because the investment in complex assets is increasing in the tax (Proposition 8). The redistributive transfers policy described in Section 5.2 also cannot implement the constrained-efficient volume of complex banks. To see this, note that the redistributive tax only affects the complex-asset price in bad times through the volume of complex banks. Therefore it suffices to check whether it can implement the price in the planner solution, which is equal to $P_C^W(b) = 1$ for $L > \hat{L}$. However, the proof of Proposition 10 shows that the price determined by the tax is strictly between 0 and 1.

□

Proof of Proposition 12

Proposition 12. *Denote by $\Delta(P_C(b)) = \mathbb{E}[U_C|P_C(b)] - \mathbb{E}[U_S|P_C(b)]$ the relative benefit of investing in complex assets without the tax as expressed in equation (4), by V^W the constrained-efficient volume of complex banks, and by $P_C^W(b)$ the complex-asset price in bad times for the constrained-efficient allocation. Then the following hold:*

- *If $L < \hat{L}$ and $\frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)} < \frac{R(1-L)-(\kappa-L)}{R(1-L)}$, then the constrained-efficient volume of complex banks can be implemented by transferring from simple to complex banks via a tax at the rate $\tau^* = \frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)}$.*
- *If $L > \hat{L}$ and $\frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)} < \frac{\mu_g R(1-L)-(L-\kappa)}{\mu_g R(1-L)}$, then the constrained-efficient volume of complex banks can be implemented by transferring from complex to simple banks via a tax at the rate $\tau^* = \frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)}$.*

Additionally, the tax level that implements the constrained-efficient volume of complex banks also maximizes welfare.

Proof. Note that the tax only affects the complex-asset price in bad times through the volume of complex banks. Therefore it suffices to check whether it can implement the price in the planner solution $P_C^W(b)$.

First consider the case where $L < \hat{L}$. For a tax level τ, complex-asset price $P_C^\tau(b)$, and volume of complex banks V^τ, the expected return for a complex bank is

$$\begin{aligned}\mathbb{E}[U_C|P_C^\tau(b), V^\tau] = \eta\Big(\phi\alpha\kappa + L - \phi\kappa + \mu_g R(1-L)\left(1 + \frac{1-V^\tau}{V^\tau}\tau\right)\Big) \\ + (1-\eta)\Big(\phi\alpha + 1 - \phi\Big)\Big(L + P_C^\tau(b)\mu_b R(1-L)\Big),\end{aligned}$$

and the expected return for a simple bank is

$$\begin{aligned}\mathbb{E}[U_S|P_C^\tau(b), V^\tau] = \eta\Big(\mu_g(\alpha\kappa\phi + L - \kappa\phi + R(1-L)(1-\tau)) + (1-\mu_g)(\alpha\phi + 1 - \phi)L\Big) \\ + (1-\eta)\left(\mu_b\Big(\phi\alpha\kappa + \frac{L-\phi\kappa}{P_C^\tau(b)} + R(1-L)\Big) + (1-\mu_b)(\phi\alpha + 1 - \phi)L\right).\end{aligned}$$

The complex-asset price is determined by the indifference condition:

$$
\begin{aligned}
0 &= \mathbb{E}[U_C | P_C^\tau(b), V^\tau] - \mathbb{E}[U_S | P_C^\tau(b), V^\tau] \\
&\quad + \eta \left[(1-\mu_g)\phi(\kappa - L)(\alpha - 1) - \frac{\mu_g R(1-L)}{V^\tau}\tau \right] \\
&\quad + (1-\eta)\mu_b \left[(\alpha\phi + 1 - \phi)(L + P_C(b)\mu_b R(1-L)) - \left(\alpha\kappa\phi + \frac{L - \kappa\phi}{P_C(b)} + R(1-L) \right) \right] \\
&\quad + (1-\eta)(1-\mu_b)(\alpha\phi + 1 - \phi) P_C(b)\mu_b R(1-L) \\
&= \Delta(P_C^\tau(b)) + \frac{\eta\mu_g R(1-L)}{V^\tau}\tau. \qquad (75)
\end{aligned}
$$

Therefore, the tax level that is consistent with the constrained-efficient price $P_C^W(b)$ and volume V^W is given by

$$
\tau^* = \frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)}.
$$

The parametric assumptions for this proposition ensure that this tax can be implemented while maintaining an equilibrium of the form described in Proposition 2. To show this, we have to check that the tax does not induce a run for banks that invest in the taxed asset.

The incentive-compatibility condition for normal depositors of simple banks to withdraw late requires

$$
\begin{aligned}
&\frac{L - \kappa\phi + R(1-L)(1-\tau)}{1-\phi} > \kappa \\
&\qquad\qquad \iff \tau < \frac{R(1-L) - (\kappa - L)}{R(1-L)}.
\end{aligned}
$$

This is satisfied by τ^* by assumption in the proposition. It is simple to show that it is incentive compatible for both simple and complex banks to continue to offer $R_d = \kappa$. Specifically, the local optima for all of the cases in the proof of Lemma 1 are unaffected by the tax as well as the inequality in (23).

It is straightforward to check that welfare only depends on the tax through its effect on the volume of complex banks and the complex-asset price in bad times. Therefore, the welfare-optimizing tax coincides with the tax that implements the constrained-efficient volume of complex banks.

The case where $L > \hat{L}$ follows analogously. $\square$

ONLINE APPENDIX—NOT FOR PUBLICATION

A Portfolio Mix

This section shows that the comparative statics of the model are qualitatively robust to allowing banks to have portfolios containing a mix of complex and simple assets. To this end, we introduce two assumptions. First, if a bank invests in a mixture of complex and simple assets, then the returns for the two types of assets are independent. This assumption preserves the fundamental distinction between complex and simple assets, which is the timing at which their returns are observed. In particular, it prevents inference of the return of a bank's complex assets at time $t = 1$ based on the observed value of its simple assets. Second, we assume that there is some level of investment in complex assets which is sufficient to trigger a run in the bad state.

The first assumption is sufficient to ensure that complex banks do not have any incentive to mix.

Proposition 13. *If the returns to the two types of assets are independent, then complex banks have no incentive to deviate, i.e., by also investing in simple assets.*

Proof. Complex banks do not have an incentive to deviate because given the conditions under which they experience a run, shifting to simple assets does not affect their expected payoff. In particular, in the good state, complex banks do not experience a run and earn an expected return of $\mu_g R(1-L)$ on their long-term assets regardless of whether they are simple or complex. Note that the bank must be sufficiently invested in complex assets in order to ensure that the expected return is sufficiently high for the late depositors, otherwise it will experience a run if its simple assets yield a low return.

In the bad state, complex banks experience a run and can generate liquidity by selling their long-term assets. If a complex bank invests a fraction A of its portfolio in complex assets and $1 - A$ in simple assets, then the total amount of liquidity it can generate in the interbank market is $P_C^*(b)\mu_b R(1-L)A + 1_{return=R} P_S^*(b) R(1-L)$. In equilibrium, the price for 1 normalized unit of complex assets $P_C^*(b)$ must be equal to the price for 1 normalized unit of simple assets $P_S^*(b)$. Therefore, the expected liquidity value is still equal to $P_C^*(b)\mu_b R(1-L)$. $\square$

The second assumption determines to what extent simple banks are willing to mix. If the level of investment in complex assets that triggers a run is sufficiently small, then simple banks may not have an incentive to invest in complex assets at all. For example, there may be fixed costs to investing in either type of long-term asset such that investing in complex assets to a small enough degree to avoid a run in the bad state is not worth it.

Even without any such fixed costs, simple banks will in general either choose to invest in complex assets up to that level or not at all. In simulations using parameters motivated by the Great Financial Crisis (as described in Section D of this Online Appendix), we find that simple banks choose to invest in complex assets up to the threshold, and that the comparative statics of $P_C^*(b)$ and V^* are similar as the threshold varies. Regarding welfare implications, we find that $\hat{L}$ decreases with the threshold, which increases the tendency for there to be too many complex banks rather than too few.

B Repo-market Interpretation

The interbank market for direct asset sales can also be interpreted as a repo market. To see this, suppose that, instead of selling assets, banks can sell assets. Denote the state-dependent price for a bond backed by complex assets by $P_C(\omega)$ and the price of a bond backed by simple assets with a high return by $P_S(\omega)$. Note that simple assets with a low return cannot be used as collateral since they are publicly observed to be worthless. The *repo rate* is the rate of return $\frac{1}{P_\theta(\omega)}$. The *haircut* $h_\theta(\omega)$ is defined as the percentage difference between the market value of collateral and the cash that is exchanged at the start of a repo. We assume that the haircut is equal to $h_\theta(\omega) = 1 - P_\theta(\omega)$, which implies that a lender holds 1 dollar of collateral for each bond purchased. In that case, the return from investing in either type of repo is equal to the repo rate, regardless of whether the borrower defaults. The rest of the model follows as in the original presentation except that the repo interpretation has the additional feature of a haircut.

C Comparative Statics of L_{min}

If the optimal tightness of liquidity requirements is the minimum level that is consistent with the parametric assumptions in Propositions 1 and 2, then the comparative statics are determined based on these restrictions.

Proposition 14. *If $\mu_b R < 1$, then the minimum level of liquidity requirements that is consistent with the parametric restrictions in Propositions 1 and 2, denoted L_{min}, satisfies the following:*

- $\frac{\partial L_{min}}{\partial \eta} \geq 0$
- $\frac{\partial L_{min}}{\partial \mu_g} \leq 0$

- $\frac{\partial L_{min}}{\partial \mu_b} \leq 0$
- *If additionally* $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} \geq 1$, *then* $\frac{\partial L_{min}}{\partial \phi} \geq 0$
- $\frac{\partial L_{min}}{\partial \kappa}$ *has the same sign as* $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} - \frac{1-\phi}{1-L}$.

The comparative statics with respect to R *and* α *depend on which constraint binds at* L_{min}.

Proof. First, we rule out parametric restrictions that do not affect the comparative statics of L_{min}. In particular, (33) does not involve L and, therefore, does not determine the comparative statics. Additionally, note that the second inequality in (15) implies that $\mu_g R > 1$. Hence, the right-hand side of (33) is decreasing in L. Hence, the second inequality in (15) is not binding at L_{min} and therefore does not determine the comparative statics. Additionally, note that the assumption $\mu_b R < 1$ implies that the left-hand side of the first inequality in (15) is increasing in L. Hence, the first inequality in (15) is not binding at L_{min} and therefore does not determine the comparative statics.

Now, suppose (14) (when the minimum is the left term) binds at L_{min}. It's straightforward to see that the right-hand side is increasing in L and α and decreasing in ϕ and that the left-hand side is increasing in R. Hence, L_{min} is increasing in ϕ and R and decreasing in α. It's straightforward to see that (14) (when the minimum is the right term) yields the same result.

Next, suppose (47) binds at L_{min}. It's straightforward to see that the left-hand side is decreasing in L and μ_g and that the right-hand side is increasing in R. Hence, L_{min} is decreasing in R and μ_g. For the remaining variables, we examine the corresponding derivatives of the left-hand side in more detail. Denote the left-hand side of (47) by LHS. For η, we have

$$\frac{\partial LHS}{\partial \eta} \propto (1-\mu_g)\phi(\kappa - L)(\alpha - 1) > 0. \tag{76}$$

Hence, L_{min} is increasing in η. For μ_b, we have

$$\frac{\partial LHS}{\partial \mu_b} \propto -\eta(1-\mu_g)\phi(\kappa - L)(\alpha - 1) < 0. \tag{77}$$

Hence, L_{min} is decreasing in μ_b. For ϕ, we have

$$\frac{\partial LHS}{\partial \phi} \propto \eta(1-\mu_g)(\kappa - L)(\alpha - 1) - (1-\eta)\mu_b(\kappa - 1)(\alpha - 1). \tag{78}$$

If $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} \geq 1$ holds, then this is positive since $L < 1$. Hence, L_{min} is increasing in ϕ. For κ, we have

$$\frac{\partial LHS}{\partial \kappa} \propto \eta(1-\mu_g)\phi(\alpha-1)(1-L) - (1-\eta)\mu_b\phi(\alpha-1)(1-\phi). \tag{79}$$

This has the same sign as $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} - \frac{1-\phi}{1-L}$. Hence, $\frac{\partial L_{min}}{\partial \kappa}$ has the same sign as $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} - \frac{1-\phi}{1-L}$. For α, we have

$$\frac{\partial LHS}{\partial \alpha} \propto \eta(1-\mu_g)\phi(\kappa-L) - (1-\eta)\mu_b\phi(\kappa-1). \tag{80}$$

If $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} \geq 1$ holds, this is positive since $L < 1$. Hence, L_{min} is increasing in α. □

To describe the intuition, this result is driven by the fact that the binding restriction at L_{min} must be (47) or (14). These restrictions ensure that the liquidity-shocked depositors always withdraw early. Specifically, (14) implies that liquidity-shocked depositors have a high utility of early consumption relative to the return of long-term investments, while (47) ensures that the equilibrium complex-asset price in bad times, $P_C^*(b)$, is greater than $\frac{1}{R}$, which can be interpreted as ensuring that the return to buying distressed assets in the interbank market is not too high.

As an illustration, Figure C.1 shows how L_{min} varies with the long-term return R and the probability of the good state η (using the parameter values from the simulation in Section D). In this case, (14) (when the minimum is the right term) is binding. As R increases, the liquidity-shocked depositors have a stronger incentive to withdraw late. This implies that L_{min} must simultaneously rise to increase the payoff from withdrawing late so as to maintain an equilibrium in which depositors withdraw early. As η increases, L_{min} is constant as long as the binding restriction is (14) (when the minimum is the right term). Eventually, (47) becomes the binding restriction. In that case, as η increases, the incentive to invest in complex assets increases due to their superior performance in the good state. This forces the equilibrium complex-asset price in bad times, $P_C^*(b)$, to decrease so as to maintain the indifference between investing in either type of long-term asset. However, if the equilibrium price is already equal to the lower bound of $\frac{1}{R}$, then L_{min} must alternatively increase to reduce the advantage of complex banks in good times. The intuition for the other parameters can be observed by following a similar pattern of reasoning.

D Simulation Motivated by the GFC

This section describes a simple simulation of the model based on parameters motivated by the Great Financial Crisis.

We select the eight parameters R, L, κ, η, μ_g, μ_b, α, ϕ to match five empirical targets.[42,43] The empirical counterparts are as follows:

- The long-term return R is selected to match 1.102, which is approximately the mean of the 30-year fixed-rate mortgage rate (1.064)[44] and the yield of high-yield bonds (1.1403) in September 2008.[45]

- The short-term interest rate $R_D = \kappa$ is selected to match 1.018, which is the federal funds rate in September 2008.[46]

- The liquidity level L is selected to match 0.179, which is approximately the ratio of total liquid assets to total assets based on 2008Q3 FR Y-9C filings for bank holding

[42] We leave three empirical features for validation.

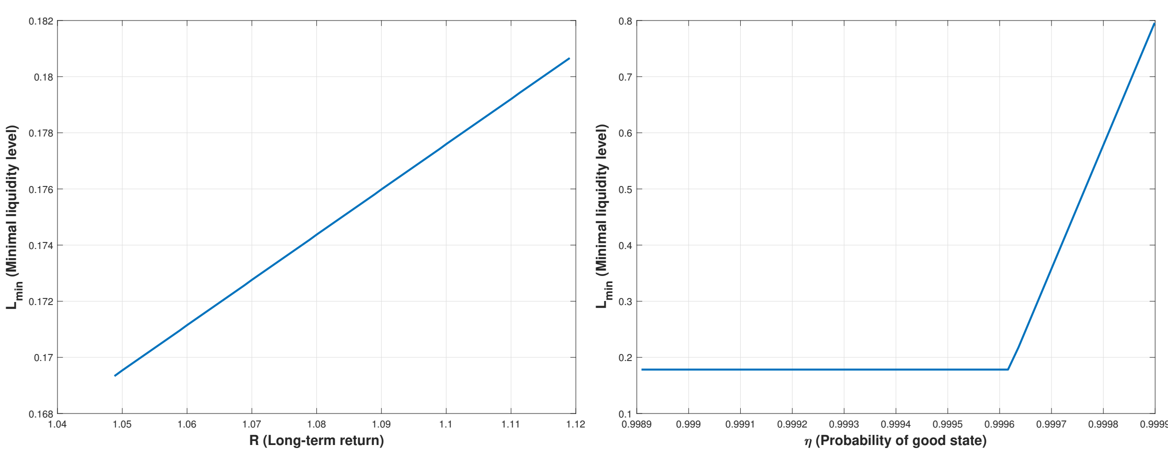
[43] Note that this parameter set satisfies the parametric assumptions mentioned in the proofs of Propositions 1 and 2.

[44] Source: Freddie Mac, 30-Year Fixed Rate Mortgage Average in the United States [MORTGAGE30US], retrieved from FRED, Federal Reserve Bank of St. Louis.

[45] Source: Ice Data Indices, LLC, ICE BofA US High Yield Index Effective Yield [BAMLH0A0HYM2EY], retrieved from FRED, Federal Reserve Bank of St. Louis.

[46] Source: Board of Governors of the Federal Reserve System (US), Federal Funds Effective Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis.

Figure C.1: Variation of L_{min}. This figure shows the minimum liquidity level that is consistent with the parametric restrictions in Propositions 1 and 2, L_{min}, as a function of the long-term return R (left) and the probability of the good state η (right). The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: R varies in the left panel, $L = 0.179$, $\kappa = 1.018$, η varies in the right panel, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$.



companies. Liquid assets include cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, and government agency debt and mortgage-backed securities (not including government-sponsored agency (GSE) debt and MBS).[47]

- The exogenous parameters are selected such that the complex-asset price in bad times $P_C^*(b)$, which is also the ratio of the complex-asset price in bad times to the complex-asset price in good times, matches 0.966, which corresponds to the ratio of the 3-month U.S. dollar London interbank offer rate (LIBOR)-OIS spread at its peak on October 10, 2008 (365 basis points, which corresponds to price of $1/1.0365 \approx 0.965$) to its level just before the onset of the GFC in the summer of 2007 (10 basis points, which corresponds to a price $1/1.001 \approx 0.999$).[48] The LIBOR-OIS spread, defined as the difference between lending rates for short-term, unsecured loans in the interbank market and the expected policy rate for overnight loans, reflects premia for both credit risk and liquidity risk (Gorton and Metrick, 2012). In particular, some movements in the LIBOR-OIS spread during the financial crisis were plausibly associated liquidity hoarding by banks and uncertainty about the distribution of non-performing assets (the Bank of England's Quarerly Bulletin (Volume 47, No. 4, 2007)). Hence, aside from lending, these liquidity concerns as well as concerns about the asset quality of potential counterparties could also lead to negative price pressure for asset purchases.

- The exogenous parameters are selected such that the fraction of complex assets V^* matches 0.139, which is the ratio of complex assets to total illiquid assets based on 2008Q3 FR Y-9C filings. Illiquid assets are defined as assets minus liquid assets, as given above. Complex assets include GSE MBS, non-agency MBS, and asset-backed securities.[49]

Table D.1 presents the selected parameters, and Table D.2 compares the empirical and model-generated values for the observables.

For validation, we also find that the selected values of $1 - \mu_g = 0.001$ and $1 - \mu_b = 0.225$, which correspond to the rate of non-performing long-term assets in good times

[47] This definition of liquid assets is an approximation for the set of (level 1) high quality liquid assets that can be used to satisfy the LCR without any discount, which includes excess reserves, Treasury securities, government agency debt and MBS (not including government-sponsored agency debt and MBS), and sovereign debt with zero risk-weights.

[48] Source: Sengupta and Tam (2008).

[49] Note that structured financial products are omitted for this exercise since banks were not required to report them at the time.

Table D.1: Selected parameters.

Parameter	Value
High return (R)	1.104
Liquidity ratio (L)	0.179
Short-term return (κ)	1.018
Probability of good state (η)	0.999
Probability of high return in good state (μ_g)	0.999
Probability of high return in bad state (μ_b)	0.775
Fraction of liquidity-shocked depositors (ϕ)	0.007
Marginal utility from liquidity shock (α)	6.23

and bad times, respectively, match the default rate of high-yield bonds before the crisis (0.5% in 2007) and at the peak during the crisis (14% in 2009).[50,51] We also find that the selected value of $\eta = 0.999$, which corresponds to the probability of the good state, matches the fraction of months in $1990 - 2007$ during which there was not a recession (0.926).[52]

Table D.2: Comparison of empirical and model-generated variables.

Variable	Empirical	Model
Matched observables		
High return (R)	1.104	1.104
Liquidity ratio (L)	0.179	0.179
Short-term return (κ)	1.018	1.018
Price in bad times ($P_C^*(b)$)	0.966	0.962
Fraction of complex assets (V^*)	0.139	0.165
Validation		
Probability of high return in good state (μ_g)	0.995	0.999
Probability of high return in bad state (μ_b)	0.86	0.775
Probability of good state (η)	0.926	0.999

The simulation yields a number of interesting observations. First, the threshold level of liquidity determining the planner solution (see Proposition 4), $\hat{L} = 0.234$, is

[50] Source: Fitch U.S. Leveraged Loan and High-Yield Default Indices.

[51] They are also close to the corresponding delinquency rates of residential mortgages, which are 2.55% in 2007 and 10.9% at the peak in 2010. Source: Board of Governors of the Federal Reserve System, Delinquency Rate on Single-Family Residential Mortgages, Booked in Domestic Offices, All Commercial Banks [DRSFRMACBS], retrieved from FRED, Federal Reserve Bank of St. Louis.

[52] Source: NBER-based Recession Indicators for the United States from the Period following the Peak through the Trough [USREC], retrieved from FRED, Federal Reserve Bank of St. Louis.

greater than the selected $L = 0.179$. Therefore, Proposition 4 implies that, perhaps surprisingly, there was underinvestment in complex assets during this time. Following the reasoning in Section 4.3, this is because the gains for the buyers in the interbank market associated with increasing the volume of complex banks, and hence decreasing the complex-asset price during the crisis, was larger than the losses for the sellers. Figure E.1 in Section E of this Online Appendix shows how the equilibrium and planner solutions vary with L. Consistent with Proposition 3, the complex-asset price increases with liquidity requirements. It turns out that the investment in complex assets also increases with liquidity requirements, indicating that the increased insurance against runs in bad times offsets the reduction in the advantage of complex assets in good times (see Section 4.2 for further elaboration of these channels affecting the incentive to invest in complex assets). Finally, the optimal liquidity tightness of liquidity requirements in both the equilibrium and the planner's solution is less than $\hat{L}$, consistent with Proposition 5.

Online Appendix F picks the parameters based on the COVID-19 crisis. We compare the GFC and the COVID-19 crisis since they are two major crises that occurred before and after the introduction of the LCR. We find that the results are qualitatively similar.

E Supplemental Figures

Figure E.1: Variation in L. This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, the planner solution, and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: $R = 1.104$, L varies on the x-axis, $\kappa = 1.018$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$.

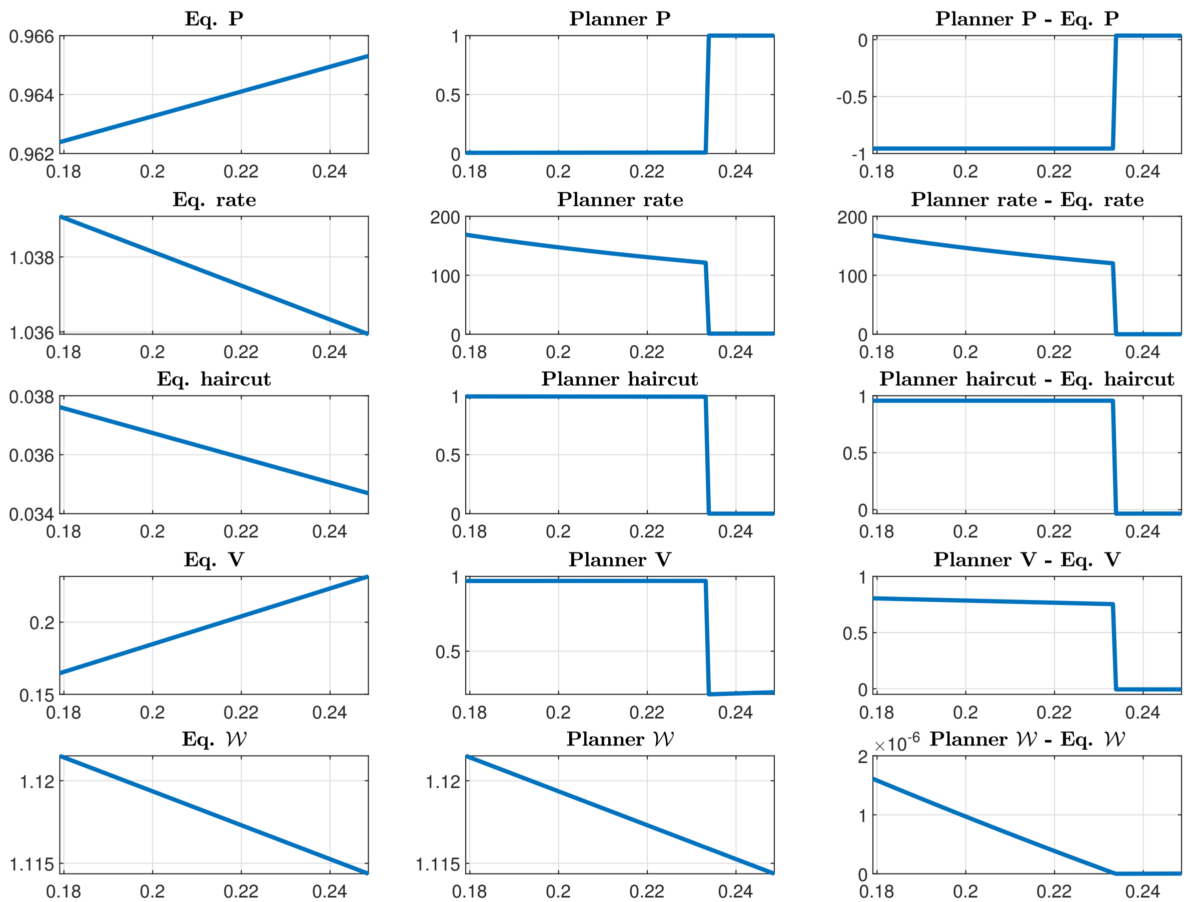


Figure E.2: Variation in L under unanticipated QE. This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, under optimal unanticipated QE, and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: $R = 1.104$, L varies on the x-axis, $\kappa = 1.018$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.

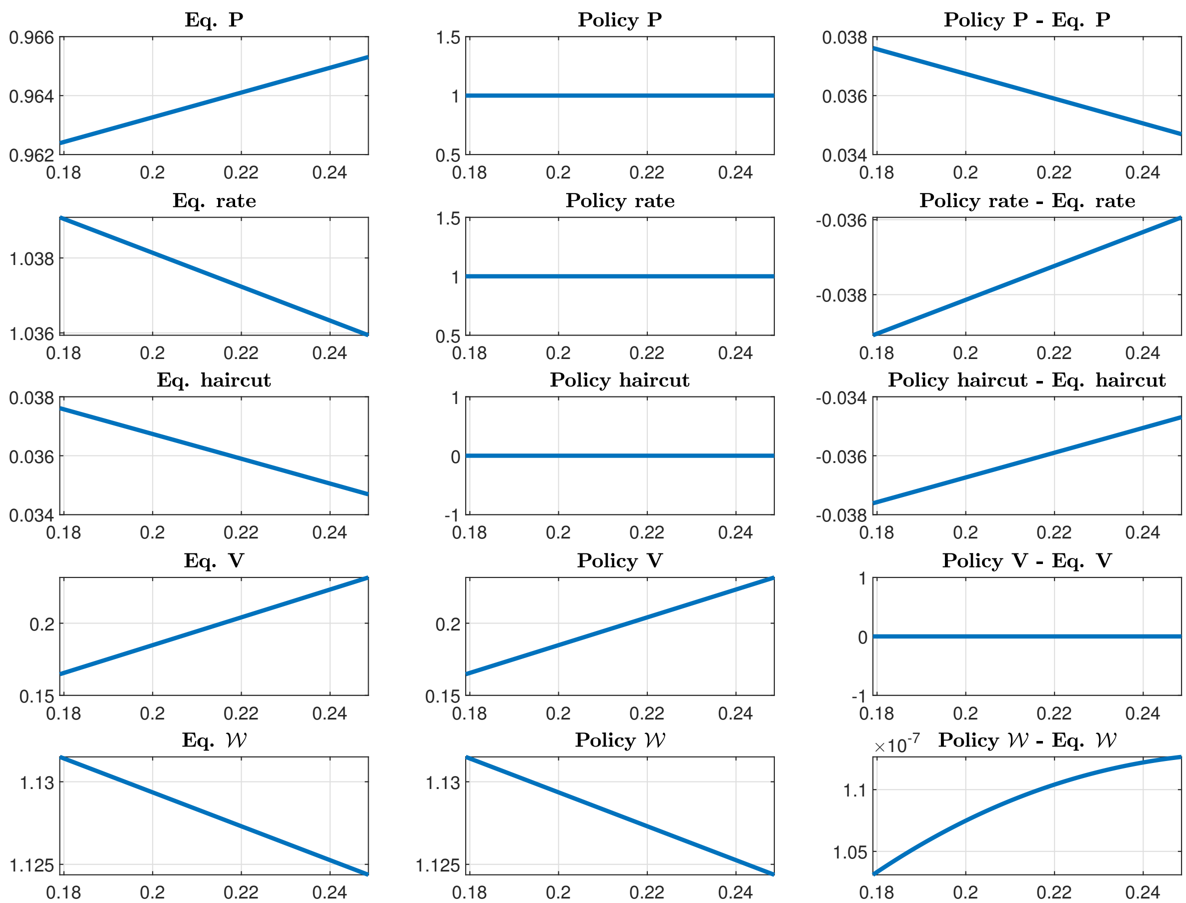


Figure E.3: Variation in L under anticipated QE. This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, under optimal anticipated QE ("Policy" or "Pol."), and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: $R = 1.104$, L varies on the x-axis, $\kappa = 1.018$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.

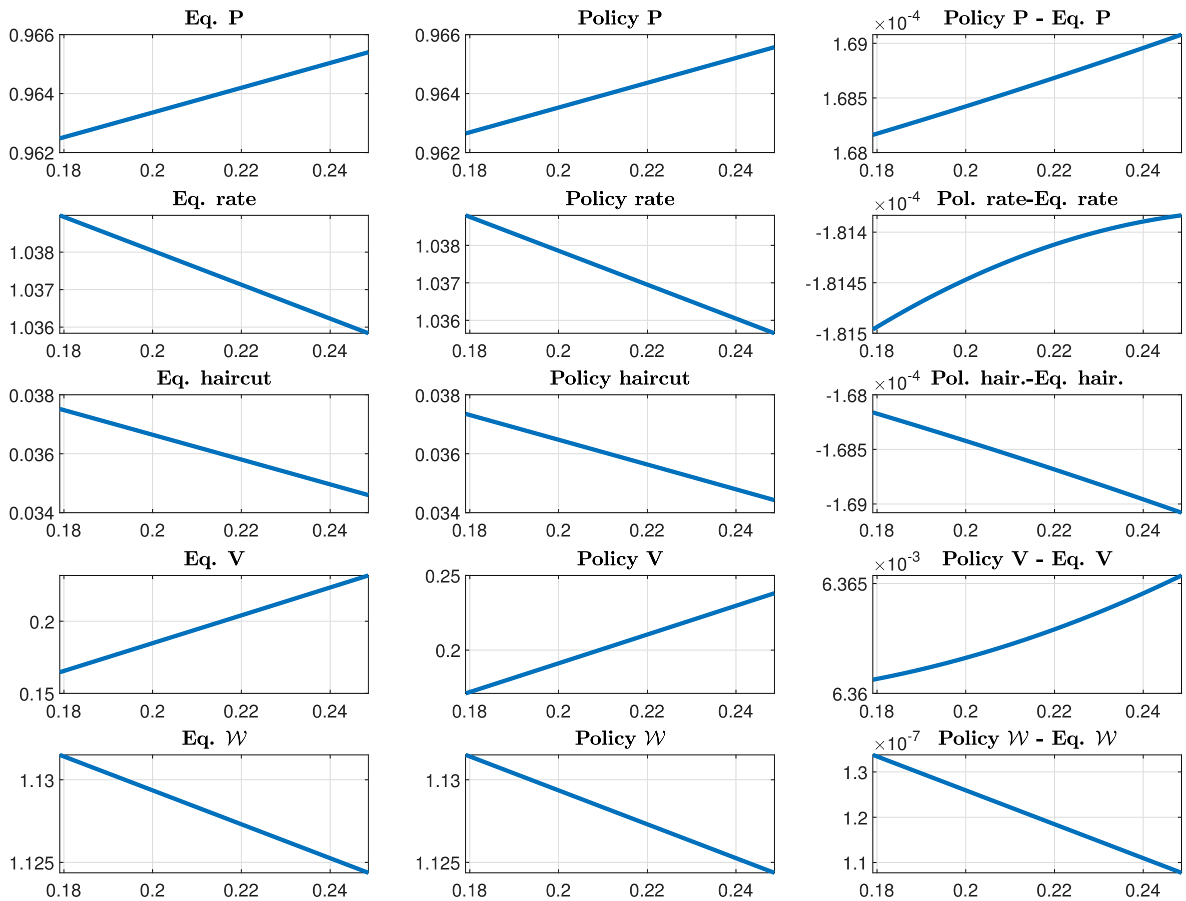
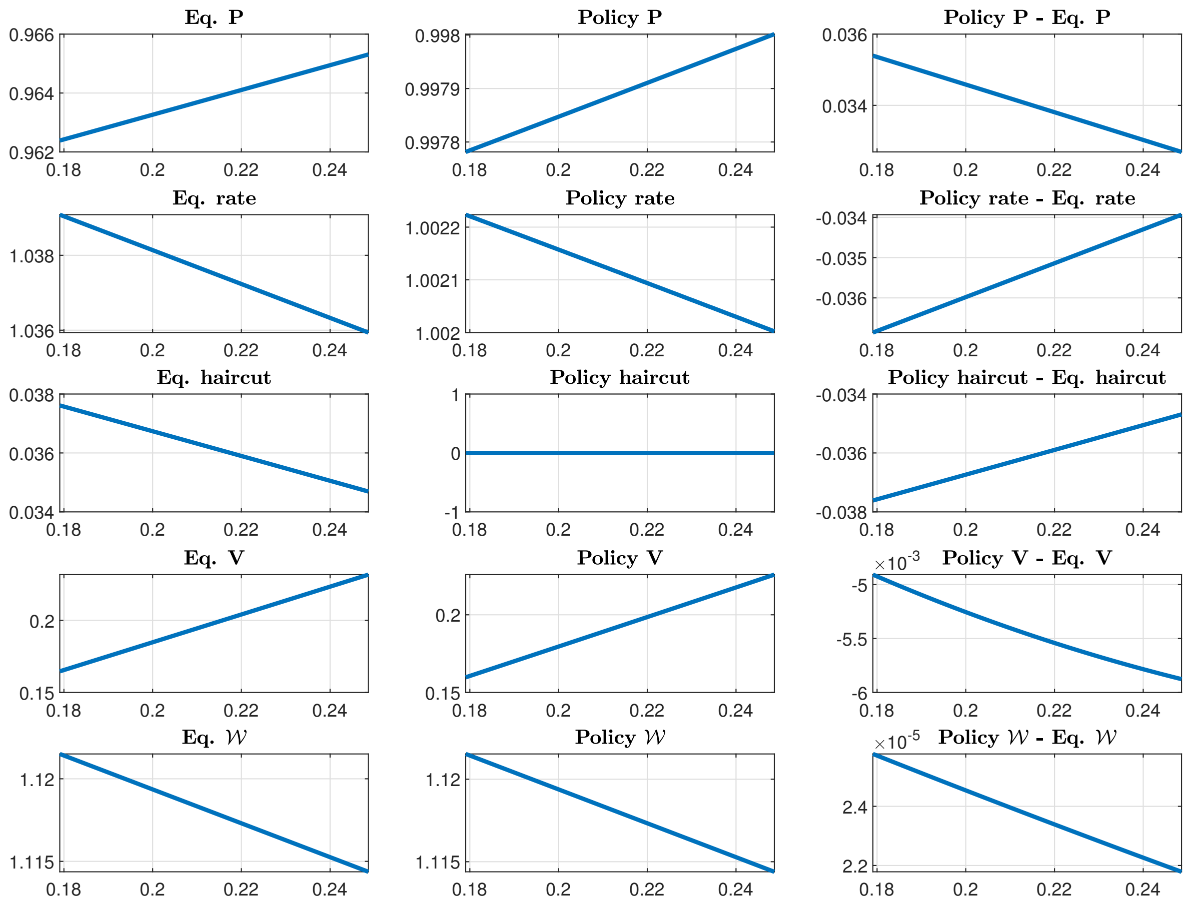


Figure E.4: Variation in L under ex-ante insurance policy. This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, under the ex-ante insurance policy, and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the Great Financial Crisis (as described in Online Appendix D) and are as follows: $R = 1.104$, L varies on the x-axis, $\kappa = 1.018$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$.



F Simulation Motivated by the COVID-19 Crisis

This section describes the results when the parameters are selected based on the COVID-19 crisis rather than the Great Financial Crisis.

We select the parameters of the model in a manner analogous to the description in Section D:

- The long-term return R is selected to match 1.063, which is approximately the mean of the 30-year fixed-rate mortgage rate in March 2020 (1.0345)[53] and the yield of high-yield bonds in March 2020 (1.092).[54]

- The short-term interest rate $R_D = \kappa$ is selected to match 1.0065, which is the federal funds rate in March 2020.[55]

- The liquidity level L is selected to match 0.203, which is approximately the ratio of total liquid assets to total assets based on 2019Q4 FR Y-9C filings for bank holding companies. Liquid assets include cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, and government agency debt and mortgage-backed securities (not including government-sponsored agency (GSE) debt and MBS).

- The exogenous parameters are selected such that the complex-asset price in bad times $P_C^*(b)$, which is also the ratio of the price in bad times to the price in good times, matches 0.988, which corresponds to the ratio of the 3-month U.S. dollar London interbank offer rate (LIBOR)-Effective federal funds rate (EFFR) spread at its peak in April 1, 2020 (138 basis points, which corresponds to a price of $1/1.0138 \approx 0.986$) to its level just before the onset of the COVID-19 crisis on February 3, 2020 (15 basis points, which corresponds to a price $1/1.0015 \approx 0.9985$).[56]

- The exogenous parameters are selected such that the fraction of complex assets V^* matches 0.157, which is approximately the ratio of complex assets to total illiquid assets based on 2019Q4 FR Y-9C filings. Illiquid assets are defined as assets minus liquid assets, as given above. Complex assets include GSE MBS, non-agency MBS, asset-backed securities, and structured financial products.

Table F.1 presents the selected parameters, and Table F.2 compares the empirical and model-generated values for the observables. Unlike the simulation of the model for the GFC (see Section D), the threshold level of liquidity $\hat{L} = 0.231$ is greater than $L = 0.203$, which implies that there is underinvestment of complex assets in equilibrium.

The remaining results are qualitatively similar to the simulation based on the GFC:

[53]Source: Freddie Mac, 30-Year Fixed Rate Mortgage Average in the United States [MORTGAGE30US], retrieved from FRED, Federal Reserve Bank of St. Louis.

[54]Source: Ice Data Indices, LLC, ICE BofA US High Yield Index Effective Yield [BAMLH0A0HYM2EY], retrieved from FRED, Federal Reserve Bank of St. Louis.

[55]Source: Board of Governors of the Federal Reserve System (US), Federal Funds Effective Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis.

[56]LIBOR source: ICE Benchmark Administration Limited (IBA), 3-Month London Interbank Offered Rate (LIBOR), based on U.S. Dollar [USD3MTD156N], retrieved from FRED, Federal Reserve Bank of St. Louis. EFFR source: Board of Governors of the Federal Reserve System (US), Federal Funds Effective Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis.

Table F.1: Selected parameters.

Parameter	Value
High return (R)	1.063
Liquidity ratio (L)	0.203
Short-term return (κ)	1.006
Probability of good state (η)	0.999
Probability of high return in good state (μ_g)	0.999
Probability of high return in bad state (μ_b)	0.775
Fraction of liquidity-shocked depositors (ϕ)	0.007
Marginal utility from liquidity shock (α)	6.23

Table F.2: Comparison of empirical and model-generated variables.

Variable	Empirical	Model
High return (R)	1.063	1.063
Liquidity ratio (L)	0.203	0.203
Short-term return (κ)	1.006	1.006
Price in bad times ($P_C^*(b)$)	0.988	0.963
Fraction of complex assets (V^*)	0.157	0.194

- Figure F.1 shows that the comparative statics with respect to L are similar to Figure E.1.
- Figure F.2 shows that the effect of unanticipated QE is similar to Figure E.2.
- Figure F.3 shows that the effect of anticipated QE is similar to Figure E.3.
- Figure F.4 shows that the effect of a redistributive policy is similar to Figure E.4.
- Figure F.5 shows that the comparison of the welfare effects of the different policies is similar to Figure 2.

Figure F.1: Variation in L (COVID-19 crisis simulation). This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, the planner solution, and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the COVID-19 Crisis (as described in Online Appendix F) and are as follows: $R = 1.063$, L varies on the x-axis, $\kappa = 1.006$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$.

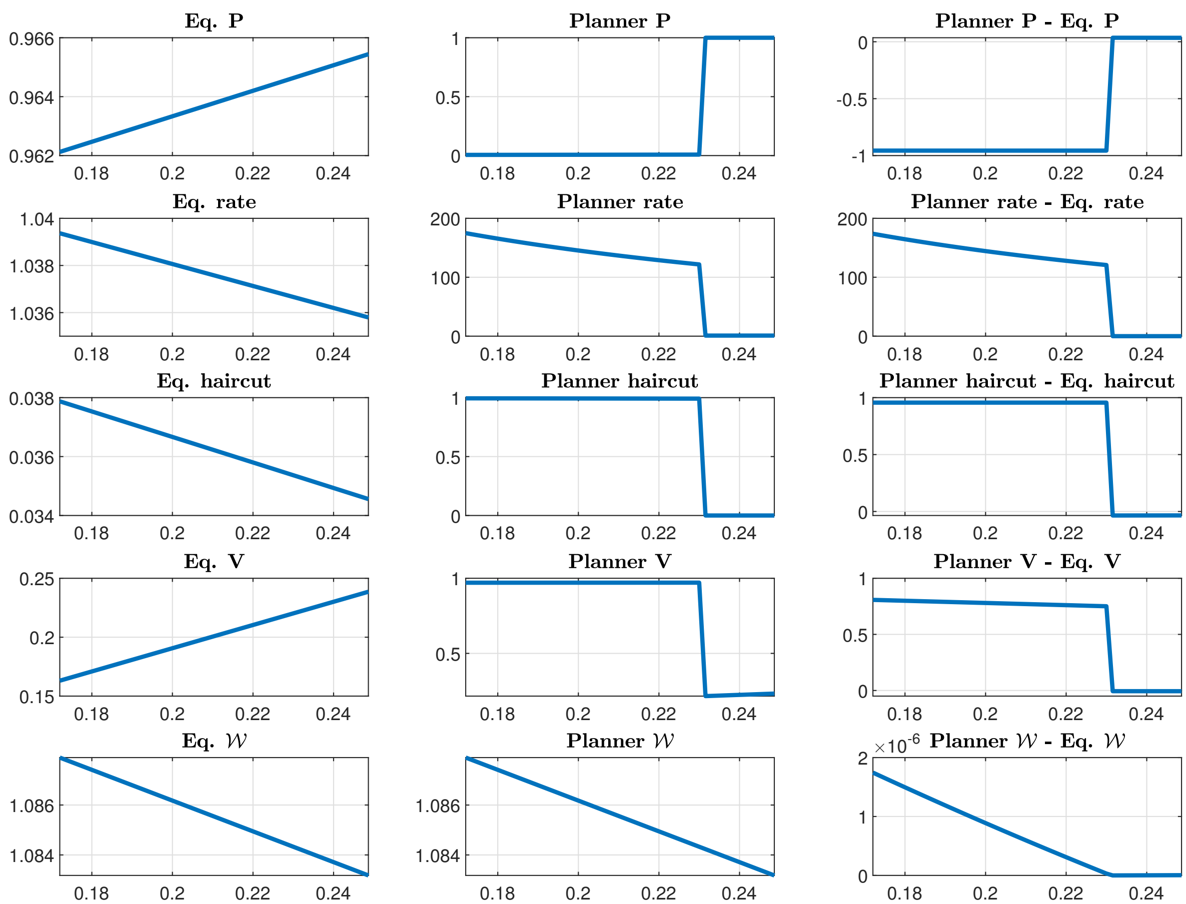


Figure F.2: Variation in L in unanticipated QE (COVID-19 crisis simulation). This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, under optimal unanticipated QE, and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the COVID-19 Crisis (as described in Online Appendix F) and are as follows: $R = 1.063$, L varies on the x-axis, $\kappa = 1.006$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.

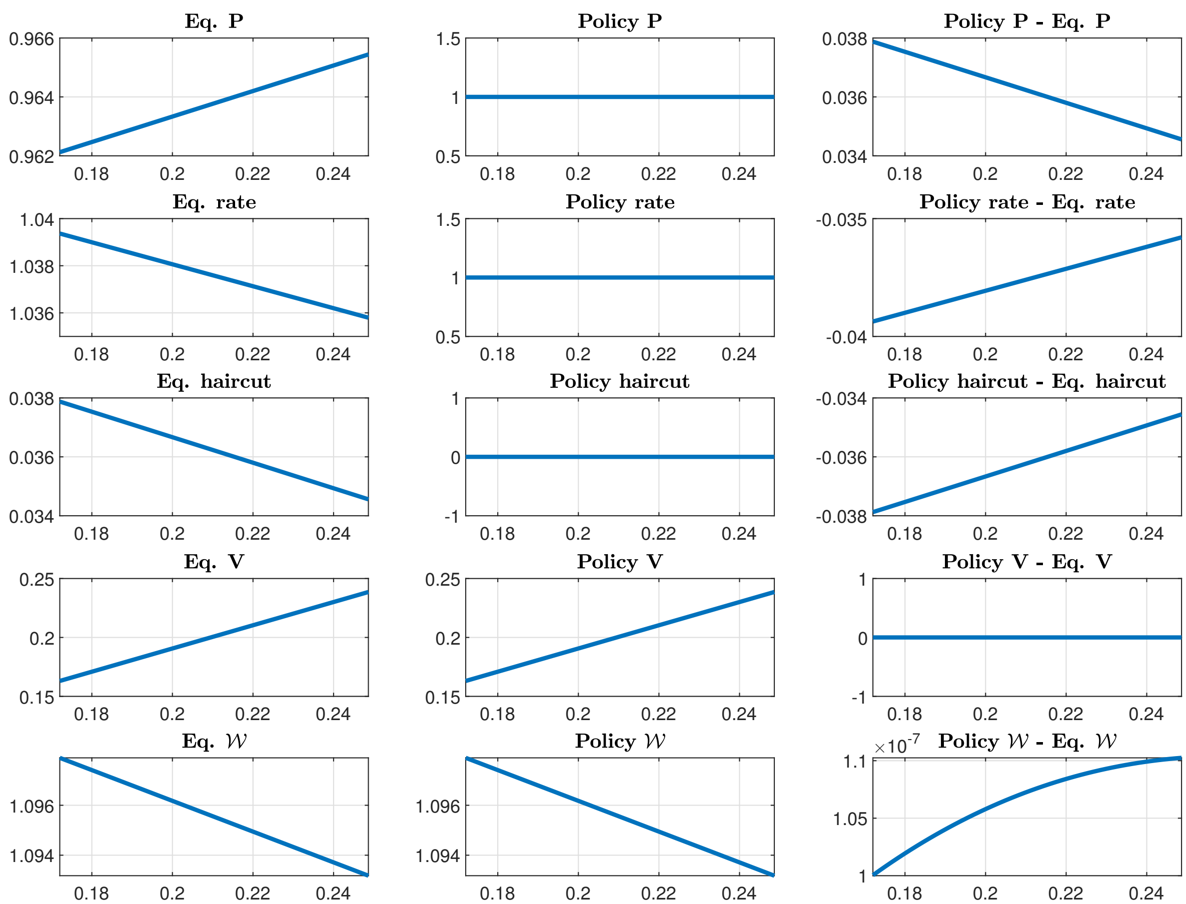


Figure F.3: Variation in L in anticipated QE (COVID-19 crisis simulation). This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, under optimal anticipated QE ("Policy" or "Pol."), and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the COVID-19 Crisis (as described in Online Appendix F) and are as follows: $R = 1.063$, L varies on the x-axis, $\kappa = 1.006$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.

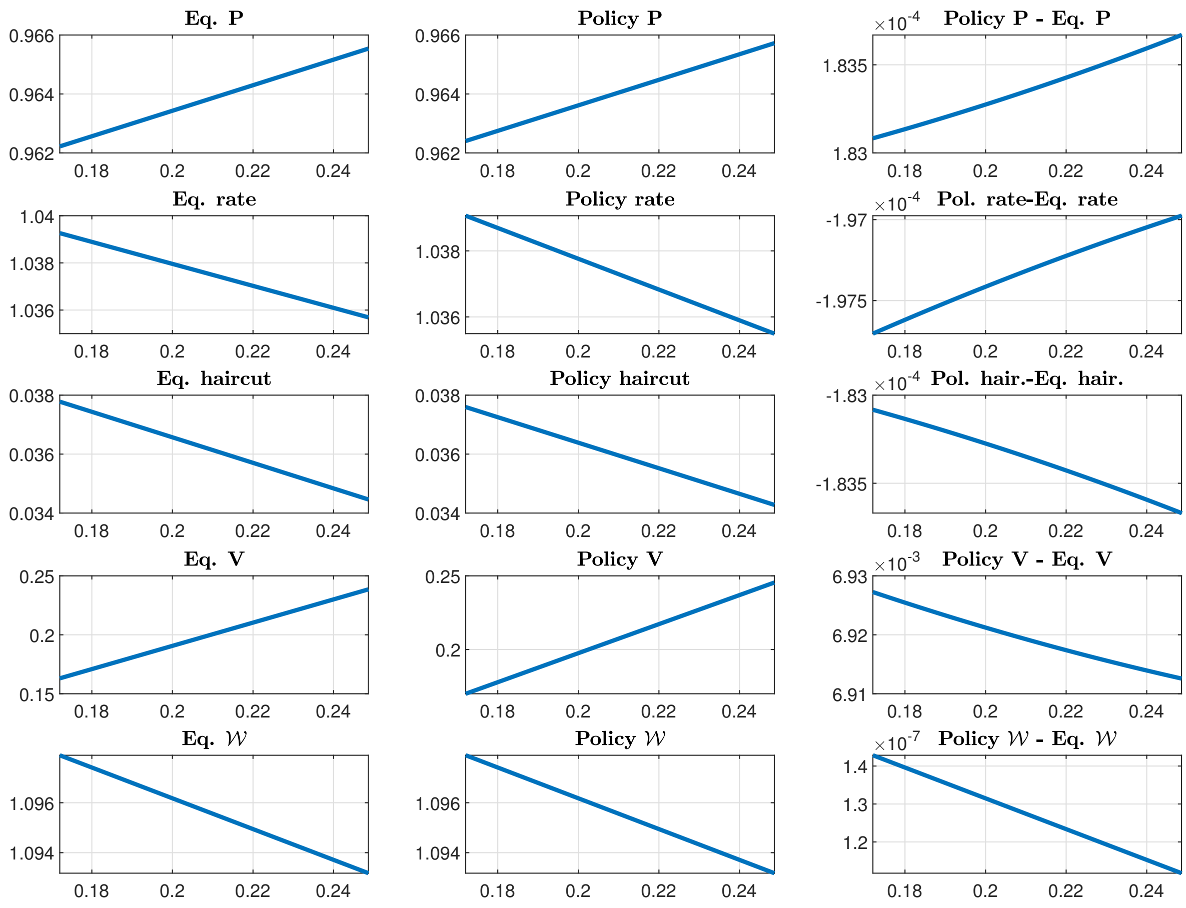


Figure F.4: Variation in L in the redistributive policy (COVID-19 crisis simulation). This figure shows, across rows and in this order, how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with L in the equilibrium solution, under the ex-ante insurance policy, and the difference between them (across columns). The haircut only applies when the interbank market is interpreted as a repo market rather than direct asset sales (see Online Appendix B). The parameters are motivated by the COVID-19 Crisis (as described in Online Appendix F) and are as follows: $R = 1.063$, L varies on the x-axis, $\kappa = 1.006$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$.

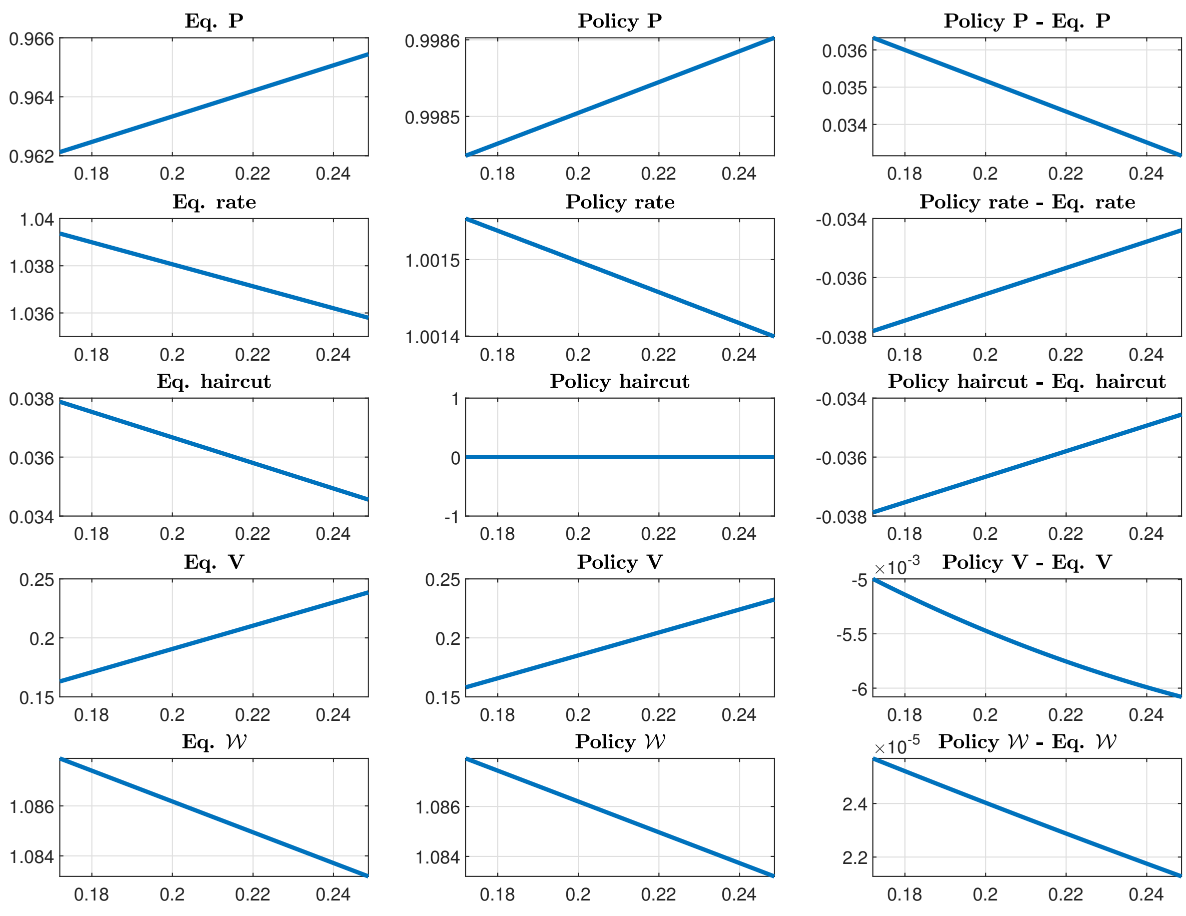
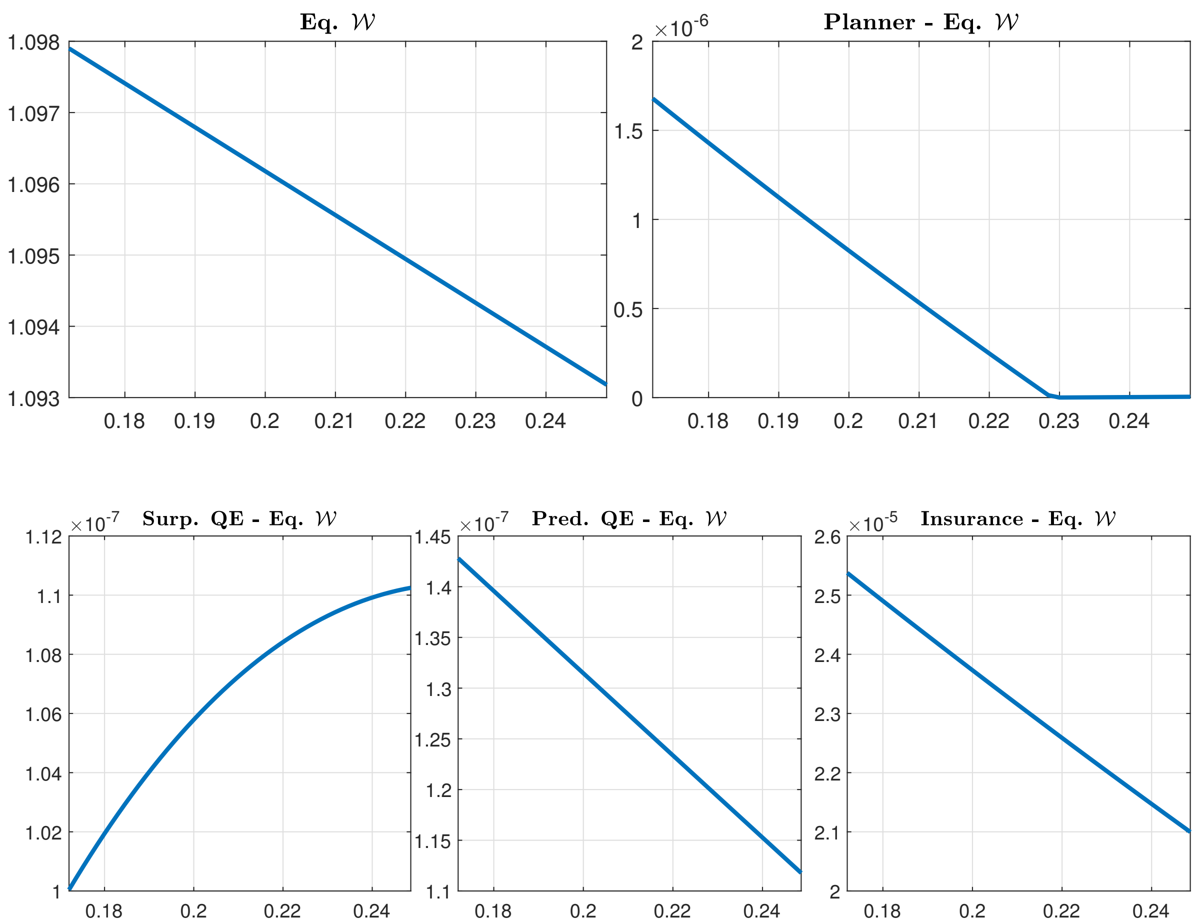


Figure F.5: Comparison of welfare gains from policy (COVID-19 crisis simulation). This figure shows welfare as a function of the liquidity level L in the baseline equilibrium with income shocks as well as the improvement in utility associated with unanticipated QE ("Surp. QE"), anticipated QE ("Pred. QE"), and the ex-ante insurance policy. The parameters are motivated by the COVID-19 Crisis (as described in Online Appendix F) and are as follows: $R = 1.063$, L varies on the x-axis, $\kappa = 1.006$, $\eta = 0.999$, $\mu_g = 0.999$, $\mu_b = 0.775$, $\phi = 0.007$, $\alpha = 6.23$. The parameters corresponding to the extension of the model with QE are $\nu = 1$ and $\delta = 0.01$.



G Empirical Evidence

This section presents evidence that the Liquidity Coverage Ratio (LCR) has been associated with higher interbank lending prices during crises by comparing the great financial crisis (GFC), which occurred before the introduction of the LCR, with the COVID-19 crisis, which occurred afterwards. It also shows that the LCR was associated with increased investment in complex assets by affected U.S. banks. Finally, it presents suggestive evidence that the LCR was associated with an amplified effect of QE on MBS prices.

G.1 The Effect of Liquidity Regulation on Interbank Lending Prices

Recall that the model shows that higher liquidity requirements are associated with higher complex-asset prices in bad times (see Proposition 3 and Figure E.1). Consistent with this result, it can be seen from Figure G.1 that the GFC in 2008, the last crisis preceding the introduction of the LCR, was associated with a more dramatic increase in the London interbank offer rate (LIBOR)-effective federal funds rate (EFFR) spread, which is a measure of concerns about credit risk and liquidity risk in short-term, unsecured interbank lending markets, compared to the COVID-19 crisis in 2020, the first crisis following the introduction of the LCR in the U.S. This is consistent with the model. However, we acknowledge that it is difficult to disentangle the effect of the LCR from that of various other differences between the two crises, such as the origins of the crises arising from either the financial system or the real economy, their magnitudes, and other policy responses.

The remainder of this section presents further evidence that is consistent with this result by comparing the 3-month U.S. dollar LIBOR to the EFFR spread during stock market corrections, which is a proxy for turbulent times in financial markets, before versus after the introduction of the LCR. Stock market corrections are periods over which the S&P 500 declines by at least 10% from peak to trough. Precise dates are obtained from Yardeni Research, Inc.

Figure G.1 shows the LIBOR-EFFR spread from January 2005 to April 2020 as well as periods with stock market corrections. Table G.1 shows the mean LIBOR-EFFR spread during stock market corrections before versus after the introduction of the LCR, as well as the t-statistic for a difference-of-means test. The average LIBOR-EFFR spread during stock market corrections experiences a drop after the introduction of the LCR that is statistically significant at the 5% level. This finding is consistent with the model, although caveats about interpreting this observation causally still apply because we cannot rule out confounding effects due to other changes in the financial system that have occurred during this time period.

Figure G.1: The LIBOR-EFFR spread. This figure shows the 3-month U.S. dollar LIBOR to effective federal funds rate (EFFR) spread from January 2000 to April 2020. Periods exhibiting stock market corrections are indicated by grey shading, and the proposal of the LCR in 2013Q4 is indicated by the dashed line. Stock market corrections are periods over which the S&P 500 declines by at least 10% from peak to trough. Precise dates are obtained from Yardeni Research, Inc.

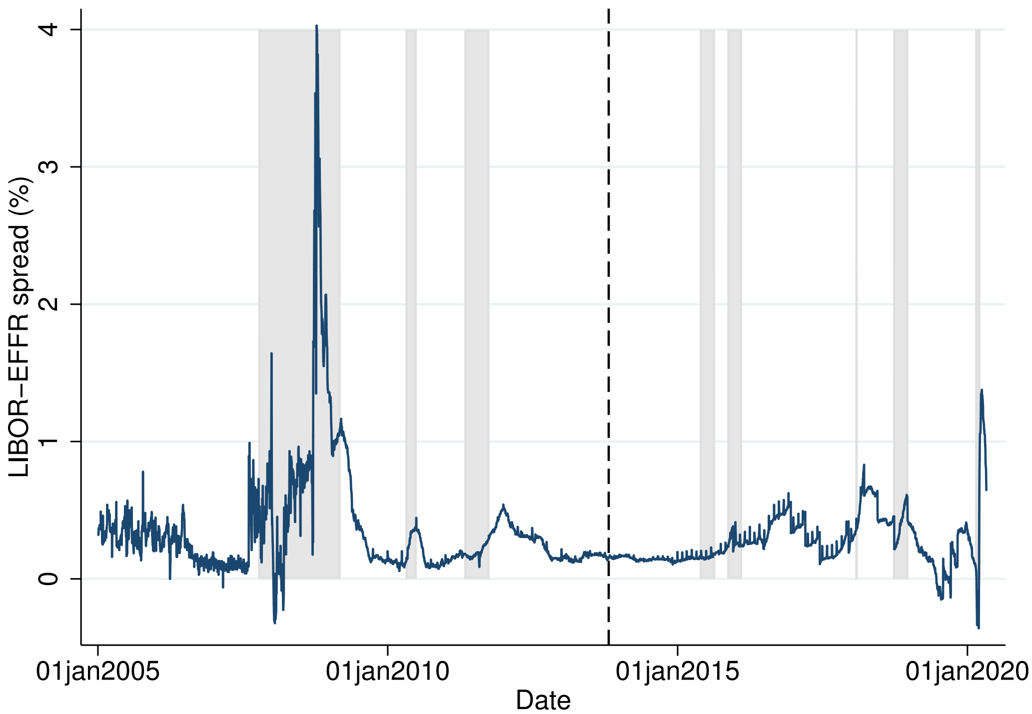


Table G.1: Average LIBOR-EFFR spread. This table shows the average 3-month U.S. dollar LIBOR to effective federal funds rate spread during stock market corrections since the year 2005 that occurred either before or after the proposal of the LCR in 2013Q4. It also shows the t-statistic from a difference in means test comparing observations before vs after the introduction of the LCR. Stock market corrections are periods over which the S&P 500 declines by at least 10% from peak to trough. Precise dates are obtained from Yardeni Research, Inc.

Before LCR	After LCR	*t*-statistic
0.709	0.270	-8.936

G.2 The Effect of Liquidity Regulation on Complex Assets

The numerical simulation of the model in Figure E.1 indicates that complexity is increasing in tighter liquidity requirements, such as those imposed under the LCR.[57] In Figure G.2, we compare the average portion of complex assets (specifically, structured financial products, asset-backed securities, and mortgage-backed securities other than those that are counted as level 1 liquid assets satisfying the LCR without any discount) out of illiquid assets (specifically, assets other than those that are counted as level 1 liquid assets satisfying the LCR without any discount, which we approximate as cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, and debt or MBS issued by government agencies) held by LCR-affected banks vs. banks that are exempt from the LCR. In particular, we restrict to a balanced sample of the FR Y-9C reports during 2012Q1-2016Q4, and to improve comparability we restrict to the set of the largest LCR-exempt banks based on total assets as of 2015Q1 to match the number of LCR-affected banks.

Prior to the implementation of the LCR, both groups exhibit parallel (declining) trends. Consistent with the numerical simulation of our model, following the implementation of the LCR and the subsequent tightening of liquidity requirements, affected banks on average increase their portion of complex assets relative to banks that are exempt from the LCR.

[57]Note that in general, the volume of complex banks can either increase or decrease with liquidity requirements, although we have found an increasing relationship in all parameter specifications that we have examined. Figure E.1 is specifically based on parameters that are motivated by the Great Financial Crisis (as described in Online Appendix D), and the effect of liquidity regulation on investment in complex assets is not a targeted outcome.

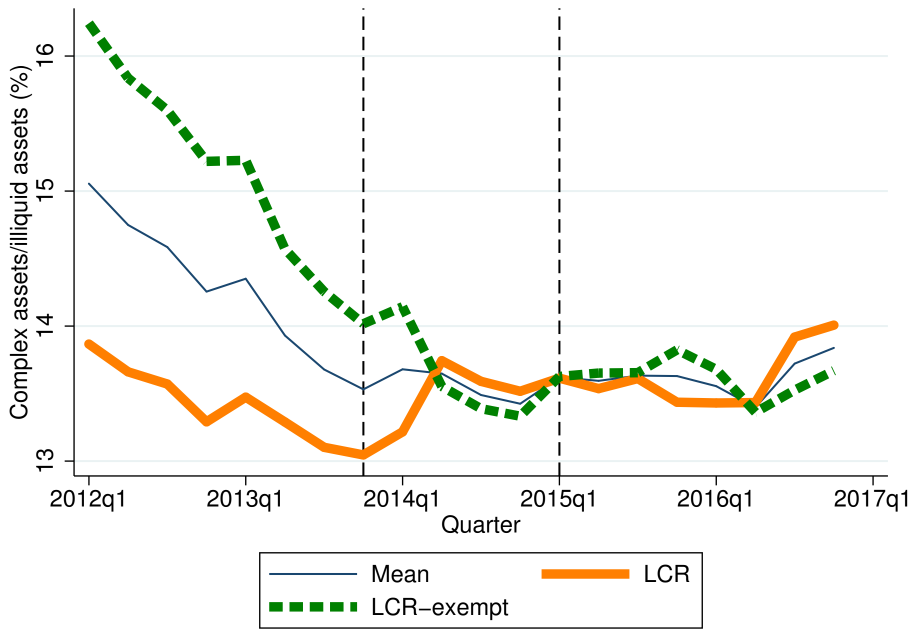


Figure G.2: The effect of the Liquidity Coverage Ratio (LCR) on holdings of complex assets. This figure shows the mean ratio of complex assets to illiquid assets, separately for all bank holding companies that were subject to the LCR and those that were exempt from it. Illiquid assets are total assets minus liquid assets, where liquid assets consist of cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, debt or MBS issued by government agencies (not including debt or MBS issued by government-sponsored enterprises). Complex assets consist of structured financial products, asset-backed securities, non-agency MBS, and MBS issued by government sponsored enterprises. The first dashed line indicates the first quarter after the proposal of the LCR (2013Q4), and the second dashed line indicates the first quarter after the implementation of the LCR (2015Q1). Data source: FR Y-9C reports, restricting to a balanced sample during 2012Q1-2016Q4 of LCR-affected banks and the same number of the largest LCR-exempt banks based on total assets as of 2015Q1.

G.3 The Effect of Liquidity Regulation on QE

Recall that the model predicts that quantitative easing (QE) increases complex-asset prices (Proposition 6 and Proposition 8). This is consistent with empirical evidence showing that QE has been associated with decreased rates of return of mortgage-backed securities (see Krishnamurthy and Vissing-Jørgensen, 2011, and Figure G.3a for the GFC, and Figure G.3b for the COVID-19 crisis). Simulations of the model show that tightening liquidity requirements can amplify the effect of QE on complex-asset prices, depending on whether it is unanticipated (Figure E.2) or anticipated (Figure E.3). This subsection presents evidence that MBS yields were more responsive to QE announcements after the implementation of the LCR compared to before the implementation of the LCR, consistent with anticipated QE.

Following the methodology in Krishnamurthy and Vissing-Jørgensen (2011), we measure the effect of QE using the change in MBS yields within a 2-day window around QE announcement dates.[58] We average yields for 15-year and 30-year current-coupon MBS backed by Ginnie Mae, Fannie Mae, and Freddie Mac.[59] To assess the effect of QE on MBS yields before the implementation of the LCR, we focus specifically on QE1 since it included purchases of MBS. We consider the same five dates as in Krishnamurthy and Vissing-Jørgensen (2011). To assess the effect of QE on MBS yields after the implementation of the LCR, we consider the QE announcements on March 15, 2020 and March 23, 2020 in response to the COVID-19 crisis. On March 15, the Federal Reserve announced that it would purchase \$500 billion in Treasuries and \$200 billion in MBS. On March 23, the Federal Reserve revised this plan, and announced that it would buy an indefinite volume of Treasuries and MBS in order to support the smooth functioning of the markets.

Table G.2 presents the findings. The average effect of QE on MBS yields was greater during the COVID-19 crisis than during the GFC, both in terms of absolute magnitude and compared to the simultaneous effect of QE on 10-year Treasury yields. This is consistent with our result that the effect of anticipated QE on the interbank complex-asset price in bad times is increasing in the tightness of liquidity requirements (Figure E.3).

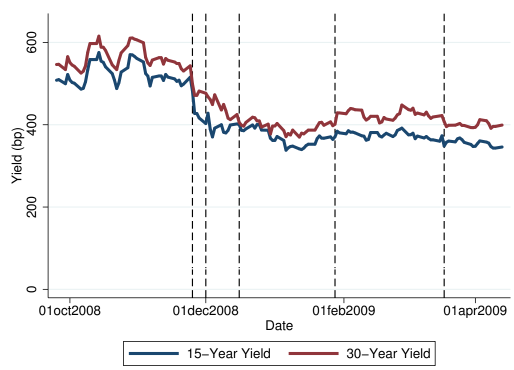
[58]Specifically, for each announcement date, we consider the difference in the last price on the trading day after the announcement date minus the last price on the trading day before the announcement date.

[59]Specifically, the 15-year yield is the average of the following MBS yield indices from Bloomberg: MTGEGNJO, MTGEFNCI, and MTGEFGCI. The 30-year yield is similarly the average of the following fields: MTGEGNSF, MTGEFNCL, MTGEFGLM.

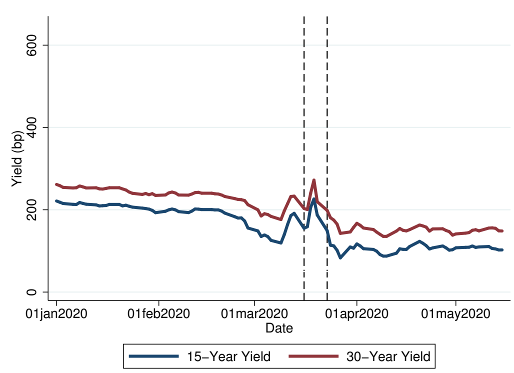
Table G.2: Effect of QE on MBS yields. This table shows the change in 10-year Treasury and 30-year and 15-year mortgage-backed securities (MBS) yields in basis points for a 2-day window around each QE announcement date. For each announcement date, we consider the difference in the last price in the trading day after the announcement date minus the last price in the trading day before the announcement date. The 15-year yield is the average of the following MBS yield indices from Bloomberg: MTGEGNJO, MTGEFNCI, and MTGEFGCI. The 30-year yield is similarly the average of the following fields: MTGEGNSF, MTGEFNCL, MTGEFGLM. The table also shows the change in each MBS yield divided by the change in the Treasury yield.

Date	10-Year Treasury	30-Year MBS	30-Year MBS / 10-Year Treasury	15-Year MBS	15-Year MBS / 10-Year Treasury
Before LCR					
Nov. 25, 2008	–36	–72	1.99	–88	2.45
Dec. 1, 2008	–25	–14	.56	12	-.46
Dec. 16, 2008	–33	–26	.78	–16	.47
Jan. 28, 2009	28	31	1.11	20	.71
Mar. 18, 2009	–41	–27	.66	–16	.38
Average	–21.4	–21.6	1.02	–17.6	.71
After LCR					
Mar. 15, 2020	–21	–30	1.43	–36	1.73
Mar. 23, 2020	–8	–39	4.85	–73	9.17
Average	–14.5	–34.5	3.14	–54.5	5.45

Figure G.3: The effect of QE announcement dates on MBS yields. Figure (a) shows 15- and 30-year yields, in basis points, of mortgage-backed securities (MBSs) around announcement dates for QE1 during the great financial crisis, as indicated by the dashed lines. The 15-year yield is the average of the following MBS yield indices from Bloomberg: MTGEGNJO, MTGEFNCI, and MTGEFGCI. The 30-year yield is similarly the average of the following fields: MTGEGNSF, MTGEFNCL, MTGEFGLM. Figure (b) similarly shows MBS yields around QE announcement dates during the COVID-19 crisis.



(a) Great financial crisis



(b) COVID-19 crisis